FERROXCUBE AERIAL RODS

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Aerial rods are, in effect, loop-aerials of greatly reduced size. The reduction in size is made possible by employing a rod of ferromagnetic material which concentrates the magnetic flux of a large area into the small loop. In this paper the effects of various parameters on the performance of an aerial rod are discussed, particular attention being given to the application of FERROXCUBE aerial rods.

INTRODUCTION

The classical loop-aerial consists of a single loop or several turns of wire enclosing a fairly large area, so as to intercept some portion of the total magnetic flux emitted by a transmitter. The efficiency of this type of aerial decreases with its dimensions, as is the case with any type of aerial. The reception obtained with small types of receivers having built-in loop-aerials tends, therefore, to be very poor. Moreover, the construction of a multi-turn loop-aerial entails a number of difficulties, so that during the last ten years engineers have searched for new principles of design.

The introduction of magnetic materials which are suitable also for high frequencies made it possible to concentrate, in a small rod, the magnetic flux from a large area, thus reducing the size of the loop to a small coil surrounding the rod. This assembly is called an aerial rod.

The design considerations leading to the most favourable construction under given conditions are not immediately evident and attempts will be made, therefore, to combine all data involved into one formula of a form which can be easily handled technically. For the sake of simplicity fairly rough approximations will be used in the first instance, after which consideration will be given to the corrections necessary for accurate design calculations.

BASIC FORMULAE

At the point of reception the field intensity and the angular frequency ω of the transmission to be received are known. Any closed loop enclosing a flux Φ of the magnetic field will have induced in it an e.m.f. of:

$$E_i = \omega\ \Phi\ n\ 10^{-8}\ \text{(volts)}, \quad . \quad . \quad . \quad . \quad (1)$$

where n denotes the number of turns of the loop. If the cross-sectional area of the loop is A cm^2 the flux $\Phi = BA$, which gives:

$$E_i = \omega\ BAn\ 10^{-8}\ \text{(volts)},$$

B being expressed in gauss. As a rule the loop aerial is tuned to the angular frequency ω by means of a capacitor, and then forms the input circuit of the receiver. The output voltage E_o across this circuit is therefore:

$$E_o = QE_i = \omega\ BAnQ\ 10^{-8}\ \text{(volts)},$$

where Q denotes the quality factor of the circuit.

The purpose of the aerial rod is to increase the flux density B at the location of the loop aerial by introducing a ferromagnetic core. This core should be more or less rod-shaped to achieve a satisfactory increase of B and to concentrate the lines of force through the turns of the coil. (It is known, for instance, that the flux density inside a plate perpendicular to the magnetizing field differs but slightly from the magnetizing force.) If the flux density B is now increased by a factor μ_{rod}, i.e. the apparent permeability of the rod-shaped core, the output voltage of the circuit is:

$$E_i = \omega\, BAn\, \mu_{rod}\, Q\, 10^{-8} \text{ (volts)} \quad . . (2)$$

It should be noted that the permeability μ_{rod} of an open magnetic circuit (e.g. in the form of a rod), is not exactly the same as the initial permeability generally quoted for ferromagnetic materials, since it is customary to measure this material constant (giving the relationship between the magnetizing force and the flux density) on a closed magnetic circuit, i.e. a toroid. To avoid confusion the "toroid permeability" will be denoted by μ_{tor} [1]).

Calculating the apparent permeability from μ_{tor} for a magnetic circuit having an air-gap of relatively small size is common practice in electrical engineering. It is true that magnetic rods are less frequently used, but the calculation of the permeability for rod-shaped magnetic cores can be found in text books [2]), and it is unnecessary to deal with this subject here.

The rod permeability μ_{rod} is a function of the toroid permeability μ_{tor} of the material used and of the physical dimensions of the rod.

Reconsidering eq. (2) it will be clear that there is more to the problem than the mere addition of a factor μ_{rod} to the product $A \cdot n \cdot Q$ of the original loop. Since this loop is part of a tuned circuit, its inductance should have a suitable value L and the introduction of the factor μ_{rod} necessarily involves a reduction of both A and n.

It will be necessary, therefore, to develop eq. (2) into an expression with L as a parameter, but before doing so it should be stressed that since the introduction of a ferromagnetic core permits reduction of A and n, it tends to decrease the effective dimensions of the loop-aerial. The reduction in size will be justified only if the rod permeability of the core material is sufficiently high. It is for this reason that FERROXCUBE, with its relatively high permeability, proves so useful.

It is important to note that the effect of the core on the inductance may differ from that produced on the concentration of the flux originating from the transmitter. In practice, the introduction of a core into a given coil increases the inductance by a factor which may be termed the coil permeability μ_{coil} and which may differ considerably from the rod permeability μ_{rod} as defined in eq. (2).

To avoid confusion it will be useful at this stage to compare the different kinds of permeability appearing in this article.

As a characteristic figure for the material the initial permeability is generally chosen. As this value is measured on a closed ring of ferromagnetic material, it is denoted in this paper by μ_{tor}. This figure is equal to the increase of the flux density in a toroidal coil when a ring-shaped core is introduced, and it is, moreover, the factor by which the inductance of the toroidal coil is increased.

Where a rod is introduced as a core the increase of the flux density in a uniform magnetizing field is called the rod permeability μ_{rod}. In this case, however, the increase of inductance of a coil wound on the rod differs considerably from the increase in flux density in the rod; this increase in therefore called the coil permeability μ_{coil}.

It is still true that the increase of the inductance is due to an increase of the flux density within the coil, but the rod is now magnetized by the coil and not by a uniform field.

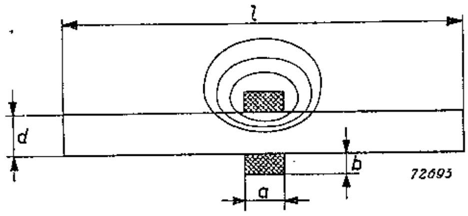

Fig. 1. Aerial rod consisting of a core of length l and diameter d, surrounded by a coil of length a and height b.

It will be shown later that the most convenient type of coil for an aerial rod is that shown in fig. 1. The inductance of such a coil, when air-cored, is given by:

$$L = n^2 d\varphi\, 10^{-8} \text{ (henrys)},$$

where φ is a constant depending on the dimensional ratios a/d and b/d of the coil (see fig. 1), as indicated in various text books [3]).

The value of the inductance is determined by the reluctance of the path followed by the magnetic flux (indicated by the thin lines in fig. 1). In an air-cored coil the ratio of the reluctance within the coil to that outside the coil may be, say, 10 : 1, which means that about 1/11 of the reluctance is

[1]) Cf. Applications and Properties of FERROXCUBE, Electr. Appl. Bull. **13**, p. 44, 1952 (No. 3/4). In this paper the symbol μ_{init} was used for the initial permeability of a closed magnetic circuit.

[2]) See, for example, R. M. Bozorth and D. M. Chapin, Demagnetizing Factors of Rods, J. Appl. Phys. **13**, p. 321, 1942.

[3]) See, for example, J. Hak, Eisenlose Drosselspulen, edit. Koehler, Leipzig.

situated outside the coil. On the other hand, when a ferromagnetic core is introduced, the reluctance of the magnetic path within the coil is negligible compared with that outside the coil. The ratio of the inductance of an air-cored coil to that of a coil provided with with a ferromagnetic core is, therefore, approximately 11 : 1. This ratio is generally independent of the permeability of the core and is determined mainly by the shape of the coil. For the type of coil shown in fig. 1, the coil permeability μ_{coil} will usually be between 5 and 15.

It is apparent, therefore, that the coil permeability μ_{coil} of a very long coil with, for example, a ratio a/d equal to 10 is comparable with the rod permeability μ_{rod}. This is due to the fact that the reluctance is determined mainly by that part of the path of the flux which is enclosed by the coil.

For the type of coil shown in fig. 1, provided with a ferromagnetic core, the inductance can be expressed by:

$$L = n^2 d\varphi\, \mu_{coil}\, 10^{-8} \text{ (henrys)}. \quad \ldots \quad (4)$$

From eqs (4) and (2):

$$E_o = \omega B Q\, \mu_{rod}\, A \sqrt{\frac{L}{d\varphi\, \mu_{coil}}} \cdot 10^{-4} \text{ (volts)}. \quad \ldots \quad (5)$$

This expression shows that a high value of E_o can be obtained by suitable choice of μ_{rod}, A and d, i.e. values which can be controlled in designing the aerial rod. First, it will be investigated what values of μ_{rod} can be obtained in practice and how they are achieved.

ROD PERMEABILITY

The rod permeability μ_{rod} of a straight rod or tube of ferromagnetic material differs from the toroid permeability μ_{tor} because in a toroid the flux density is substantially the same at all points in the anulus whereas, in a rod or tube, the flux density, due to fringing, decreases towards the poles. In a rod, therefore, the flux density initiated by an external magnetizing force, is less than that in a toroid and this effect is often called "demagnetization by open ends".

It is comparatively simple to calculate the effect of a rod on a uniform magnetic field if the rod is ellipsoidal. Most data given in text books are based on the assumption that an ellipsoid approximates closely to a rod. In this paper use has been made of the data contained in the paper quoted in footnote [2]), in which allowance has been made for the use of straight rods.

When a rod is placed in a uniform field B, the field is distorted in the vicinity of the rod (see fig. 2). The flux density thus obtained is at a maximum in the centre of the rod and decreases towards the ends. The rod permeability μ_{rod} is the ratio of the maximum flux density to the original value of B.

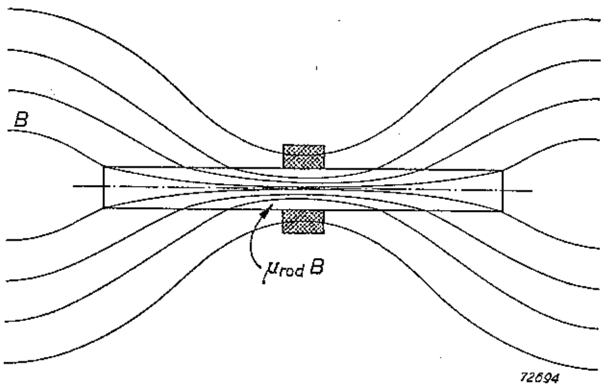

Fig. 2. Trend of the lines of force B of a uniform magnetizing field in the vicinity of the aerial rod.

The distribution of the flux density over the length of the rod is almost parabolic for long rods whose permeability μ_{rod} is fairly large. This flux density B can be measured by placing the rod in a magnetic field at some distance from a coil excited by a signal generator, and reading the voltage induced into a small search coil moved to successive positions over the rod. Fig. 3 shows the voltage induced in such a coil at a distance x from the mid-point of a rod having a total length l. The rod was obtained by extrusion from FERROXCUBE IVB, the length l being 200 mm and the diameter d being 8 mm. Measurements showed that $\mu_{rod} = 100$.

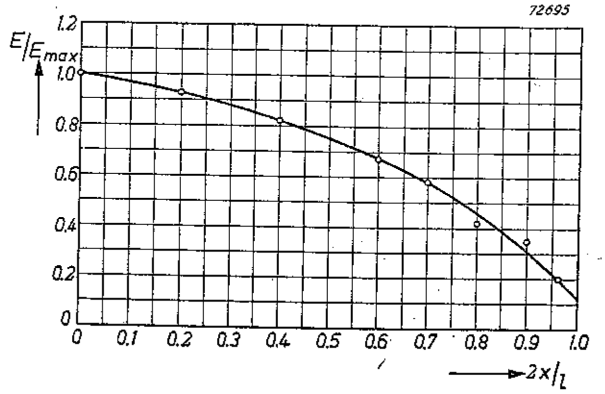

Fig. 3. Relative e.m.f. E/E_{max} induced in the coil of an aerial rod as a function of its relative distance $2x/l$ from the mid-point of the rod.

The curve of fig. 3 is applicable for most practical cases. The top of the parabola will, however, be flattened for small values of μ_{rod} or when saturation occurs.

Fig. 3 shows clearly that eq. (2) is valid only when the turns n of the coil are concentrated at the middle of the rod where the flux density is in actual fact equal to $\mu_{\mathrm{rod}}\,B$. If these turns are spread out over any considerable length of the rod the mean value of the flux density over that length can be obtained from fig. 3 by integration.

Fig. 4 gives the averaging factor f_{a}, i.e. the ratio of the mean value to the maximum value of the flux density, as a function of that section a/l of the rod surrounded by the coil. This curve was derived from fig. 3 by integration. The averaging factor f_{a} of a coil surrounding the entire length of the rod is 0.7. For a short coil $f_{\mathrm{a}} = 1$.

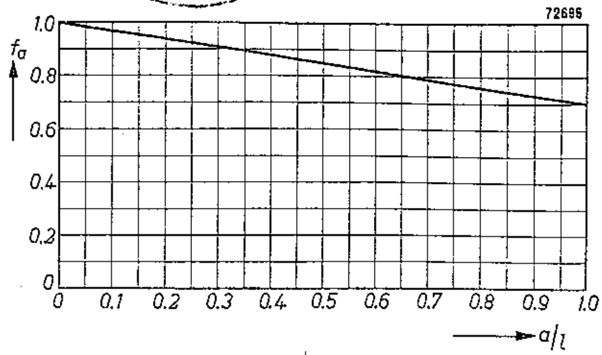

Fig. 4. Averaging factor f_{a} as a function of the ratio a/l.

As previously mentioned, the rod permeability μ_{rod} (as defined in eq. (2)) is a function of the toroid permeability μ_{tor} and the ratio l/d of the rod. The value of μ_{rod} has been plotted in fig. 5 as a function of the ratio l/d, with μ_{tor} as parameter. As might be

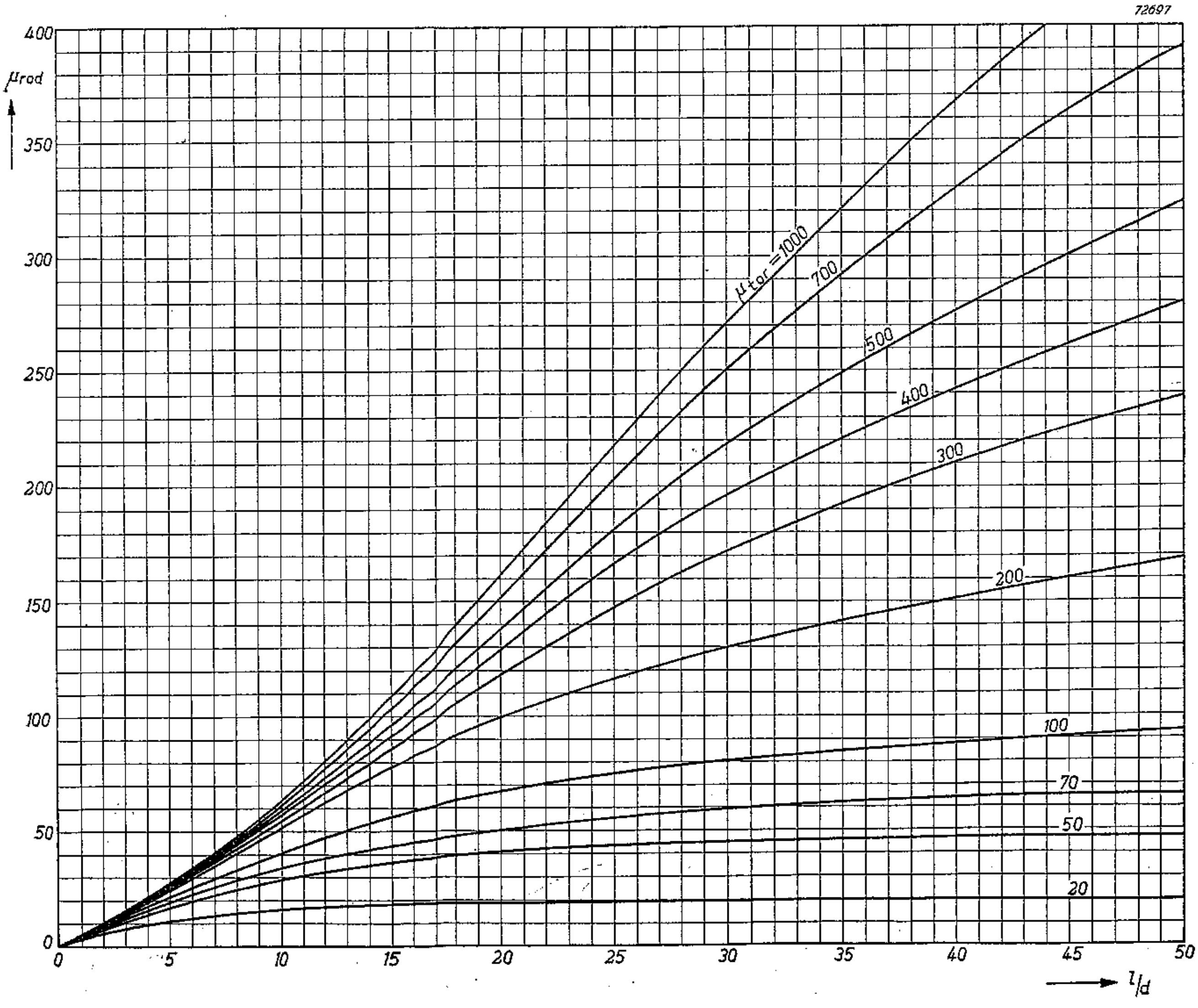

Fig. 5. Rod permeability μ_{rod} as a function of the ratio l/d with the initial permeability μ_{tor} of a toroidal core as parameter.

expected, a high value of μ_{rod} can be obtained only when the toroid permeability μ_{tor} is fairly high. Moreover, the ratio l/d must also be sufficiently large. On the other hand, if the value of the ratio l/d is limited by constructional considerations, there is little point in using a material with a high value of μ_{tor}.

Provided the value of μ_{tor} is not too small, μ_{rod} may as a first approximation be taken as roughly proportional to the ratio l/d, hence:

$$\mu_{rod} = a\, l/d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From eqs (5) and (6):

$$\boxed{E_o = \omega\, BQ\, a\, l\, \frac{\pi}{4} \sqrt{\frac{dL}{\varphi \mu_{coil}}} \cdot 10^{-4}\,(\text{volts}).} \quad . \quad . \quad . \quad (7)$$

In this expression the cross sectional area A of the rod has been replaced by $d^2\pi/4$.

Eq. (7) may be considered as the basic design formula for aerial rods. The main conclusion to be drawn from this expression is that, for a given volume of magnetic material, the aerial rod should be made as long as is possible within the constructional limit of the whole design, unless the ratio l/d becomes so large that any further increase would produce but little effect on the rod permeability μ_{rod}.

In certain instances it may be suggested to use a material with a high value of μ_{tor}, thereby resulting in a higher value of a in eqs (6) and (7). From fig. 5 it is clear that this will serve no useful purpose unless the ratio l/d is fairly large.

It follows from the comments in the preceding section that the use of a t u b e of FERROXCUBE instead of a r o d has little effect on the value of the inductance of the coil. However, it is necessary to reconsider eq. (6).

Fig. 6 shows, by way of comparison, a rod and a tube of ferromagnetic material of the same outside dimensions. When the rod (fig. 6*a*) is subjected to an external magnetizing force H, magnetic elements of the material will to a certain degree be aligned in the direction of H. The resultant of all molecular magnetizing currents within the material, therefore, produces the same flux density as would a certain number of ampere-turns round the outer surface of the rod. The flux density B_o within the rod is determined mainly by the quantity of these equivalent ampere-turns and the ratio l/d of the rod.

With the tube shown in fig. 6*b* the effect will be much the same, except that there is a second surface at the inside of the tube, and the combined effect of all the molecular currents produces the same results as a second imaginary winding. Within the bore of the tube the flux density B_o originating from the imaginary ampere-turns on the outside is cancelled out by the flux density B_i due to the imaginary ampere-turns on the inside.

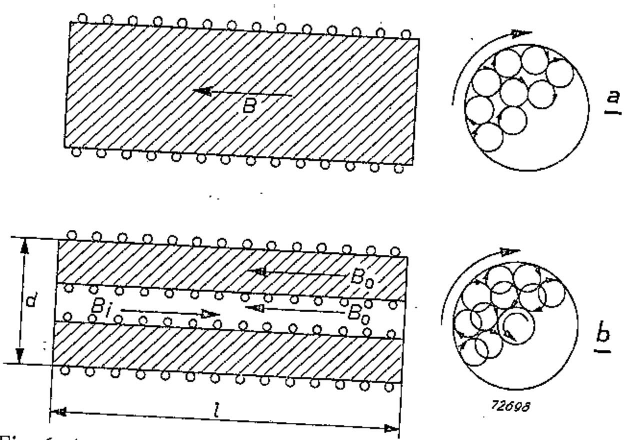

Fig. 6. Apparent ampere-turns in a solid rod of ferromagnetic material (*a*) and in a tubular rod (*b*). In the cross sections at the right the magnetizing currents of the elementary magnets and the resultants of these currents are schematically indicated.

The flux density within the material is to all intents and purposes determined by the outer ampere-turns only, as is the case with a rod. This is because the flux density caused by the inner ampere-turns is very weak at this point, due to the fact that most lines of force return via the infinite space outside the material. It can be concluded, therefore, that there is very little change within the material itself if the rod is hollowed out to form a tube, the flux density within the material remaining equal to that of the original rod. However, in the hollow part of the rod, the flux density will be equal only to that of the external magnetizing force.

To calculate the flux density in a tube it is therefore necessary first to determine μ_{rod} from fig. 5, starting from the outside ratio l/d, after which the actual cross-sectional area of the tube should be calculated and used for A in eq. (2).

It will thus be clear that, from the electrical point of view, it is more favourable to compress a tube into a solid rod of the same length but smaller outside diameter, as the active cross section A then remains unchanged, and μ_{rod} increases due to the higher ratio l/d.

However, if the toroid permeability μ_{tor} of the material is fairly small so that the ratio l/d has little effect on the value of μ_{rod}, the effect of a tube will be roughly the same as that of a rod of the same weight.

The value of μ_{rod} or of μ_{tor} of a given rod can be checked experimentally by an inductance measurement. If a single-layer coil with a fairly large ratio l/d (for example $l/d = 10$) is filled up with a core having the same length as the coil, the core will be magnetized by an approximately uniform field, and the distribution of the flux will be as shown in fig. 3. The inductance of such a coil will, therefore, increase in a ratio:

$$\frac{L_2}{L_1} = f_a\,\mu_{rod}\ ,$$

where f_a is again the averaging factor corresponding to the flux distribution over the length of the rod. The toroid permeability μ_{tor} of the material can be determined from the value of μ_{rod} thus measured by means of the graph of fig. 5.

An experimental rod of FERROXCUBE IVB with $l = 200$ mm and $d = 8$ mm was wound with 98 turns of wire over its entire length. A similar coil, wound on a glass former in place of on a rod of FERROXCUBE, was tuned to 7.7 Mc/s by means of a capacitor of 82 pF. At the same capacitance the coil with FERROXCUBE core was tuned to 0.9 Mc/s, therefore:

$$\mu_{coil} = f_a\,\mu_{rod} = \frac{L_2}{L_1} = \left(\frac{7.7}{0.9}\right)^2 = 72\ .$$

Since f_a may be taken as approximately 0.7 for a coil surrounding the entire length of the rod, the rod permeability $\mu_{rod} = 100$. This is in close agreement with the curve for $\mu_{tor} = 200$ at $l/d = 25$ (see fig. 5).

When carrying out the measurements mentioned above, it is important that the turns of the test coil are wound fairly close together. The same rod of FERROXCUBE wound with 35 turns only, spaced over its entire length, was found to have a coil permeability of 34 instead of 72.

It should be noted that, if for practical reasons the internal diameter of the coil is greater than the core diameter, the inductance L_1 of the coil without core will be determined by the cross section A_1 of the coil tunnel, whereas the inductance L_2 of the coil with a core is determined mainly by the cross section A_2 of the core, in which case the rod permeability is given by:

$$\mu_{rod} = \frac{L_2}{L_1}\frac{A_1}{A_2}\frac{1}{f_a}\ . \qquad \qquad (8)$$

It is also possible to verify by experiments the trend of the curves in fig. 5. A rod of FERROXCUBE IVB with $l = 180$ mm and $d = 4.4$ mm was fitted with a small coil of 35 turns located at its mid-point. The rod was placed in the magnetic field at some distance from a signal generator. After the coil had been tuned to the signal frequency the voltage across it was measured. Subsequently the signal generator was detuned so that the reading obtained was lower by a factor $\sqrt{2}$, thereby enabling the quality factor Q of the circuit to be determined. The voltage E_i induced in the coil was then equal to the first reading divided by the value of Q.

This process was repeated several times, 10 mm being removed from both ends of the rod for each measurement. The circlets in fig. 7 show the result of these measurements.

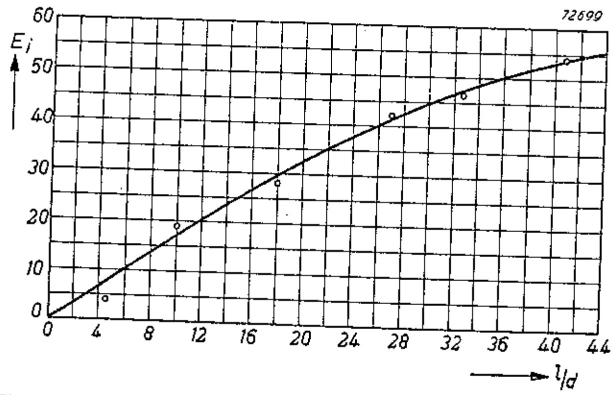

Fig. 7. Induced voltage E_i in the coil as a function of the ratio l/d.

Since the curve for $\mu_{tor} = 200$ in fig. 5 gives $\mu_{rod} = 153$ for the full length of the rod ($l/d = 41$), this value should be proportional to an induced voltage $E_i = 54$ V in fig. 7. The whole curve $\mu_{tor} = 200$ in fig. 5 can now be reduced to the scale shown in fig. 7 by multiplying all values of μ_{rod} by 54/153 (i.e. the drawn curve in fig. 7).

QUALITY FACTOR

The quality factor Q of the aerial circuit is important not only because it determines the output voltage of the loop-aerial as expressed by eq. (7), but because the circuit is required to have a reasonable selectivity. The quality factor of the circuit is determined mainly by: the resistance of the coil; the iron losses of the rod; the radiation losses and other dimensional effects in the rod; and also by the external parallel damping which is independent of the type of circuit considered. These effects are included in the following expression:

$$\frac{1}{Q} = \sum \tan\,\delta = \frac{r}{\omega L} + \tan\delta_f + \tan\delta_d + \tan\delta_p\,. \quad (9)$$

The four terms in the right-hand side of the equation represent the different types of losses. The fourth term will be disregarded here, since it has no bearing on the design of the earial.

Theoretically, the first term, $r/\omega L$, representing the copper losses, can be reduced to a minimum by using a suitable type of litz wire. On the other hand, such a design may involve a high value of $\varphi\ \mu_{coil}$ (cf. eqs (4) and (7)). A high value of μ_{coil} certainly reduces the copper losses, since the number of turns necessary to obtain the required value of L is also reduced. The problem then is to find the most favourable combination of $r/\omega L$, φ and μ_{coil}.

The second term in eq. (9), representing the iron losses, depends to a large extent on the coil permeability μ_{coil}. If the tangent of the loss angle of a closed toroid with permeability μ_{tor} is $\tan\delta$, the tangent of the loss angle δ' of the same toroid with a small air-gap and an effective or coil permeability μ' is given by [4]):

$$\tan\delta' = \tan\delta\,\frac{\mu'-1}{\mu_{tor}-1} \approx \tan\delta\,\frac{\mu'-1}{\mu_{tor}}\,. \quad (10)$$

This expression also applies very closely to a long rod surrounded along its entire length by a coil, provided μ_{coil} is taken as μ'. Although for short coils $\tan\ \delta_f$ generally decreases with decreasing μ_{coli}, eq. (10) does not apply rigorously in that case.

Fig. 8 gives the ratio $\tan\ \delta/\mu_{tor}$ as a function of the frequency for several types of FERROXCUBE. It is sufficient to multiply the values thus found by the value of $\mu' - 1$ for the type of FERROXCUBE in order to find the value of $\tan\ \delta'$ in eq. (10). At frequencies of about 1 Mc/s the value of $\tan\ \delta/\mu_{tor}$ of FERROXCUBE IVA and IVB is about 10^{-4}. If a rod of such material is used, surrounded by a coil along its entire length, where $\mu_{coil} = 100$, the iron losses will be:

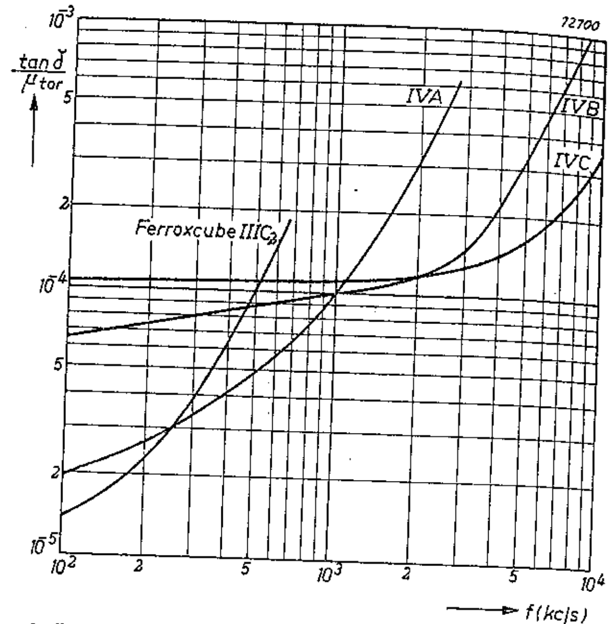

Fig. 8. Ratio $\tan\ \delta/\mu_{tor}$ as a function of the frequency f for various types of FERROXCUBE.

$$\tan\ \delta_f = 10^{-4} \times 100 = 10^{-2}.$$

This means that the quality factor of such an aerial would never exceed a value of 100 even if no other losses were present. It is necessary, therefore, to base the design on a lower value of μ_{coil}. Low values of μ_{coil} are usually obtained with short coils, and although eq. (10) no longer applies for such coils, it will be seen that values in the order of $\tan\ \delta_f = 40 \times 10^{-4}$ may be obtained, thereby leaving a margin for other losses.

The calculation of $\tan\ \delta_f$ for short coils will not be fully investigated here, but experiments show that this value depends on the size of the core, or on that part of the core which is magnetized, rather than on the value of μ_{coil}. This point will be illustrated later.

The effect of the third term in eq. (9) cannot be readily calculated, nor can it be easily determined by measurement. As with the capacitive type of aerial, the magnetic type will radiate power when oscillating, but since the loss factor due to this radiation is more or less proportional to the ratio of the dimensions to the wavelength, the radiation losses of an aerial rod will be very low. The so-called dimensional losses, which are related to the high product of permittivity and permeability, are difficult to estimate and it is, moreover, not

[4]) H. Van Suchtelen, Introduction to the Application of FERROXCUBE, Electr. Appl. Bull. 11, p. 27, 1950 (No. 2).

practicable to measure them separately from the normal iron losses, these also being dependent on the dimensions. It is, however, possible to calculate $\tan \delta_f$ with reasonable accuracy, and the values thus obtained agree so closely with actual measurements, that $\tan \delta_d$ may be neglected here.

It will now be of interest to investigate the magnitude and the behaviour of the terms in eq. (9) as measured on a random sample.

When measuring the quality factor of an aerial rod, matters may be so arranged that the parallel losses $\tan \delta_p$ can be disregarded. This can be achieved by coupling the coil, which is tuned by a calibrated variable capacitor, fairly loosely to a sensitive tube voltmeter and by making the coupling with the signal generator as loose as is convenient.

The signal generator is adjusted to the frequency at which the quality factor is to be measured, say to 1 Mc/s. Denoting the reading of the capacitor by C and the detuning required for reducing the output voltage to $1/\sqrt{2}$ of its original value by ΔC, then:

$$\frac{1}{Q} = \tan \delta = \frac{\Delta C}{C}.$$

Having determined the value of the capacitance which will produce resonance in a coil without a core, the value of C will then be a measure of μ_{coil}. (In so doing, the values of C should be fairly high to minimize the effect of the self-capacitance of the coil. In some cases it may be preferable to work with a constant capacitance C and to compare the resonant frequencies with and without core.)

The quality factor of the coil previously mentioned, having 98 turns, without core was found to be:

$$\frac{1}{Q} = \frac{r}{\omega L_1} = 200 \times 10^{-4}.$$

It was established that the same coil having a core, $\mu_{\text{coil}} = 72$, so that the copper losses with core may be expected to be:

$$\frac{r}{\omega L} = \frac{200 \times 10^{-4}}{72} = 3 \times 10^{-4}.$$

The measured total losses of the coil with core appeared to be $1/Q = 76 \times 10^{-4}$, thus leaving 73×10^{-4} as the sum of the second and third components in eq. (9). This is in reasonably close agreement with the curves of fig. 8, and it can be concluded that the third component is small.

Measurements carried out on the same rod wound with only 35 turns over its entire length showed that the coil permeability $\mu_{\text{coil}} = 34$, while the total iron losses now appear to be $\tan \delta_f = 50 \times 10^{-4}$. This shows that there is a marked discrepancy from the proportionality with μ_{coil}. The discrepancy even increases when the 35 turns are gradually concentrated towards the middle of the rod.

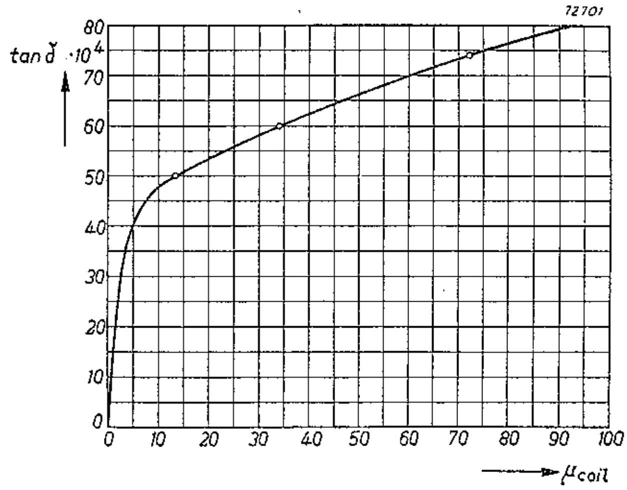

Fig. 9. Measured total losses $\tan \delta$ as a function of the coil permeability μ_{coil}.

Fig. 9 shows the total losses $\tan \delta$, including the copper losses, as a function of the coil permeability μ_{coil}. The curve shows clearly that over a wide range $\tan \delta$ depends but little on μ_{coil} and is principally determined by the quantity of core material coupled to the coil.

A second experiment was carried out on a rod of FERROXCUBE IVB provided with a coil with $a = 5$ mm consisting of 30 turns of litz wire $3 \times 7 \times 0.07$ mm. The original length of the rod of 200 mm was gradually reduced by removing short pieces from both ends. The full-line curve shown in fig. 10 gives the measured value of $\tan \delta$ as a function of μ_{coil}.

Now the trend of this curve suggests a formula of the general form:

$$\frac{1}{Q} = \frac{r}{\mu_{\text{coil}}\, \omega L_1} + \tan \delta_0 \,(\mu_{\text{coil}} - 1) \; . \quad . \quad . \; (11)$$

The first term, in which L_1 represents the inductance without core, needs no further explanation; the second term expresses a proportionality of $\tan \delta_f$ with $(\mu_{\text{coil}} - 1)$, to some extent on the lines of eq. (10).

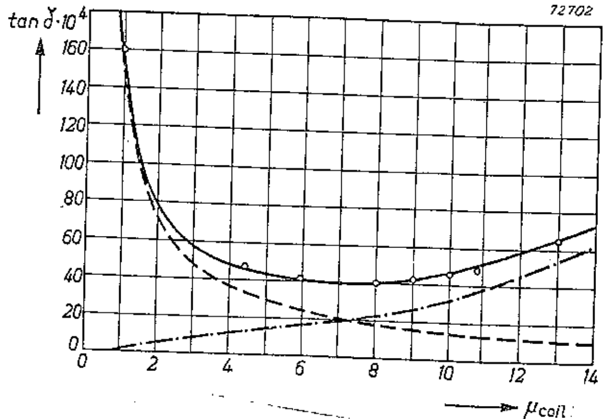

Fig. 10. Full-line: total losses $\tan\delta$ as a function of μ_{coil}, measured by gradually reducing the length a of the rod. Broken line: hyperbola representing the copper losses $r/\mu_{coil}\omega L_1$. The dash-dot line gives the difference between these curves and represents the total iron losses.

If the full-line curve in fig. 10 is a true graphic representation of eq. (11) it implies that, at its minimum, $r/\mu_{coil}\omega L_1$ should be equal to $\mu_{coil} \tan\delta_0$. Taking this assumption as a starting point, the hyperbola for $r/\mu_{coil}\omega L_1$, representing the copper losses, has been reconstructed in fig. 10 and plotted as a broken line. When these values are subtracted from the values measured, the total iron losses are left (dash-dot line). The behaviour of the iron losses approximates fairly closely to what might be expected from calculations. (These calculations will be published in a subsequent paper.)

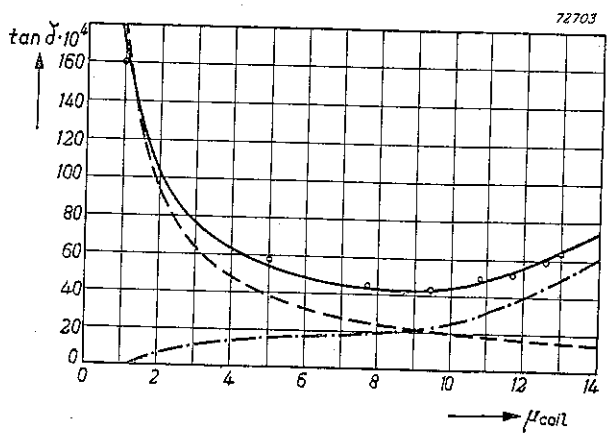

Fig. 11. As fig. 10; the variation of μ_{coil}, however, is obtained by moving the coil along a rod of given length.

Another method of varying the permeability μ_{coil} of the small coil used above is to move it from the mid-point towards one end of the rod. In this way $\tan\delta$ can again be plotted as a function of μ_{coil}, as is shown in fig. 11.

The form of the curve shown in fig. 11 is very similar to that in fig. 10. The hyperbola representing the copper losses may also be reconstructed. Similar to fig. 10, low values of $\tan\delta_f$ may be obtained. This is, in fact, quite logical as, due to the displaced position of the coil, a portion of the rod is only loosely coupled to the coil. The displacement of the coil is equivalent, therefore, to shortening the rod used in the former experiment.

Attempts can finally be made to decrease $\tan\delta_f$ by giving the coil an internal diameter greately in excess of the diameter of the rod. A coil with an internal diameter of 22 mm surrounding a rod having a diameter of 8 mm had an inductance only 1.2 times that of a closely fitting coil with the same number of turns. The total losses of this coil were $\tan\delta = 60 \times 10^{-4}$, from which the copper losses can be subtracted.

As may be seen from the hyperbola of fig. 10, the copper losses of the small coil with $\mu_{coil} = 13$ are about 12×10^{-4}. The copper losses of the larger coil are therefore:

$$\frac{r}{\omega L} = \frac{12 \times 10^{-4}}{1.2} \cdot \frac{22}{8} = 28 \times 10^{-4},$$

thus leaving for the iron losses $\tan\delta \approx 30 \times 10^{-4}$.

This proves that even by reducing the copper losses to a minimum, the total quality factor would not show an appreciable improvement.

It may therefore be concluded from the above argument and in particular from the curve given in fig. 9, that a low value of μ_{coil} is less favourable than might be expected from eq. (10). For the design of a high-quality aerial rod the reduction of the coil permeability is, therefore, by no means a general substitute for the use of high-grade ferromagnetic material. There is, however, another reason for reducing the coil permeability, as will be shown in the following section.

TEMPERATURE COEFFICIENT

Because of the temperature coefficient of FERROXCUBE the resonant frequency of a circuit containing a core of FERROXCUBE will be dependent on the temperature. If the variation of μ_{coil} per degree centigrade is denoted by $\Delta\mu_{coil}$, the relative variation, in the case of a toroid with a small air gap, will be:

$$\frac{\Delta\mu_{coil}}{\mu_{coil}} = \frac{\Delta\mu_{tor}}{\mu_{tor}} \frac{\mu_{coil} - 1}{\mu_{tor}} \quad . \; . \; . \; (12)$$

Since the resonant frequency is inversely proportional to the square root of L, the relative change in frequency is equal to half the change in μ_{coil}. Hence:

$$\frac{\Delta f}{f} = \frac{1}{2} \frac{\Delta \mu_{tor}}{\mu_{tor}^2} \cdot (\mu_{coil} - 1) \,. \quad . \quad (13)$$

The values of $\Delta \mu_{tor}/\mu_{tor}^2$ are known for the various types of FERROXCUBE [5]) and are $< 10 \times 10^{-6}$ and $< 15 \times 10^{-6}$ respectively for FERROXCUBE IVA and IVB.

If the maximum permissible frequency drift $\Delta f/f$ is taken as 1000 c/s per Mc/s, i.e. 1/1000 for a change in temperature of 30 °C, then $\Delta f/f = 1/30\,000$ per degree centigrade, so that for FERROXCUBE IVB the coil permeability will be:

$$\mu_{coil} - 1 = \frac{2 \cdot \frac{1}{30\,000}}{15 \times 10^{-6}} = 4.4 \,.$$

For rods the temperature coefficient is higher than the value given by eq. (12), so that the reduction of μ_{coil} is less effective than might be expected. It is, however, clear that μ_{coil} can not be in the order of 100, as would be the case with a very long coil.

EFFECTIVE HEIGHT

Although it is logical to express the net result of an inductive aerial by means of eq. (2) and the conclusions derived therefrom, this method fails when it is desired to compare this result with that of a capacitive aerial. It is common practice to express the field intensity of the transmitter in volts per meter, and the performance of the aerial in terms of its "effective height".

The e.m.f. induced in a capacitive aerial by an electrical field strength of F volts per meter is:

$$E_i = h \cdot F \,, \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where h denotes the effective height.

There is a fixed relation between the magnetic field B and the electric field F of a transmitter, so that it is also possible to calculate the induced voltage from F for a loop-aerial, the effective height again being defined by eq. (14). The effective height of a loop aerial can be expressed amply, by means of the formula:

$$h = \frac{2\pi A n}{\lambda} \, 10^{-4} \text{ (metres)} \,. \quad . \quad . \quad . \quad (15)$$

where A denotes the mean area of the loop in cm² and λ is the wavelength in m. It will be seen that, for a coil wound on a rod having a permeability μ_{rod}, eq. (15) becomes:

$$h = \frac{2\pi A n \, f_a \, \mu_{rod}}{\lambda} \, 10^{-4} \text{ (metres)} \,. \quad . \quad . \quad . \quad (16)$$

The effective height of a reasonably good indoor aerial is in the order of 1 m. If h is calculated for a loop of 10 turns with A = 25 cm × 25 cm, eq. (15) becomes:

$$h = \frac{2\pi \, 625 \times 10}{300} \, 10^{-4} = 0.013 \text{ m}$$

for a frequency of 1 Mc/s. This value may seem to be extremely low compared with the effective height of a capacitive aerial, but it should be remembered that the loop forms part of the input circuit of the receiver, whereas a capacitive aerial is loosely coupled to this circuit. This fact accounts for an extra reduction factor of 10 or 20 when using a capacitive aerial, so that the performance of the loop-aerial will be only 1 : 10 or 1 : 5 compared with that of an indoor aerial.

In the following section the design is based on eq. (7), but the overall performance of the design is expressed in a manner which is more familiar to the radio engineer, that is, in the product of h and Q.

DESIGN CONSIDERATIONS

It follows from the above comments that there are conflicting conditions for a high value of Q and a good concentration of the flux of the external field. In a design which aims at a maximum output voltage (E_o in eq. (7)) the value of Q may be found to be rather poor. On the other hand, if a high value of Q is aimed at, the output voltage may be well below that of the first design. It is therefore impossible to give general rules for the design of the "ideal aerial rod" and this paper will therefore be confined to showing the influence and interaction of the many parameters.

First, a design will be considered for obtaining the maximum **output voltage.** This will be based on a rod of FERROXCUBE IVB of 8 mm × 200 mm, comprehensive data of which are now available.

[5]) See the paper quoted in footnote [1]).

The ratio l/d of this rod is 25; according to the curve from $\mu_{tor} = 200$ in fig. 5, the value of μ_{rod} can further be increased by increasing the ratio l/d. If the size of the cabinet is large enough a longer rod may therefore be used to great advantage without affecting the value of Q appreciably,

If a small coil is used, situated at the mid-point of the rod, the quality factor will be approximately $Q = 10^4/60 = 167$. The permeability of a coil with $a = 5$ mm is $\mu_{coil} = 13$. In the case in point it was considered that temperature conditions allowed of a higher value of μ_{coil}, so that the length of the coil could, to advantage, be increased. The increase of μ_{coil} is then outweighed by the decrease of φ in eq. (3), so that the product $\varphi \cdot \mu_{coil}$ in eq. (7) is smaller. A coil with $a = 20$ mm proved to have the required inductance at 27 turns.

It is now possible to calculate the overall performance of the aerial. The permeability of a rod of 200 mm length is $\mu_{rod} = 115$, while the averaging factor of the coil used is almost equal to unity. At a frequency of 1 Mc/s this gives the effective height as:

$$h = \frac{2\pi \times 27 \times \frac{\pi}{4} \times 0.64 \times 115}{300} \cdot 10^{-4} = 0.0033 \text{ m}.$$

This is approximately 1/4 of the effective height found for the large loop-aerial considered in the previous section. Taking into account the difference in dimensions, this can be considered as being very good.

The overall performance of this aerial rod, expressed by the product of h and Q, is:

$$h \cdot Q = 0.0033 \times 167 = 0.55.$$

The ratio l/d of a rod of the same diameter but 300 mm long would be 38, whilst the rod permeability would be 145 instead of 115, thus giving an overall performance of:

$$h \cdot Q = \frac{145}{115} \cdot 0.55 = 0.70.$$

Fig. 12 shows the effect of lengthening the coil. Here again a rod of 200 mm × 8 mm has been wound with a constant number of turns (35) and L was measured as a function of the length of the coil. It follows from eq. (4) that L is proportional to the product $\varphi \cdot \mu_{coil}$, and may be calculated from the data available.

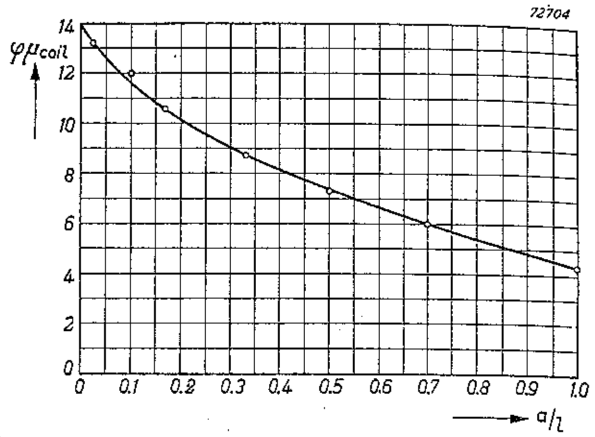

Fig. 12. Measured values of $\varphi\mu_{coil}$ as a function of the ratio a/l.

Fig. 12 shows that there is not much point in increasing the ratio a/l beyond 0.5, as the price for the decrease of $\varphi \cdot \mu_{coil}$ would be a considerable increase of μ_{coil}, i.e. higher losses and a higher temperature coefficient. In fact, even at a ratio of 0.5 the gain in overall performance compared with a coil having a ratio a/l of 0.025 is only $\sqrt{13/7.5} = 1.3$. This shows once again that, as a rule, there is little advantage to be gained in using long coils. On the other hand, as previously shown, a very narrow coil with a large diamter is also not to be recommended.

The design of an aerial whose **quality factor** has a predetermined value of, say $Q = 200$ at 1 Mc/s, will now be investigated.

Again, basing the design on a rod of 8 mm × 200 mm and a coil of $a = 5$ mm, the coil is first moved towards one end of the rod in order to attain the desired value of Q. From fig. 11 it will be seen that $Q = 200$ (i.e. $1/Q = 50 \times 10^{-4}$) can be obtained with a coil permeability $\mu_{coil} = 11$. Assuming that this value of μ_{coil} is not unsuitable in view of the temperature coefficient, the position of the coil for $\mu_{coil} = 11$ can now be determined.

Fig. 13 shows the value of μ_{coil} as a function of $2x/l$, i.e. the displacement of the coil from the mid-point of a rod 8 mm × 200 mm. This graph shows that the position of the coil should be about $2x/l = 0.6$ for the required coil permeability of 11.

The decrease of the coil permeability must be compensated by a corresponding increase of the number of turns, in this instance by a factor $\sqrt{13/11} = 1.1$.

The displacement of the coil results in a decrease of the enclosed flux (see fig. 3). For $2x/l = 0.6$ this decrease of B is seen to be 0.63.

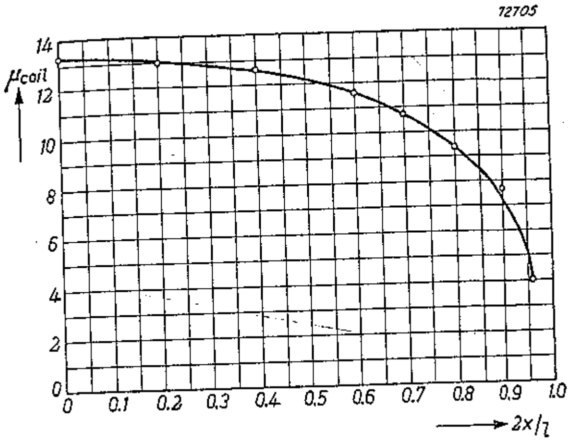

Fig 13. Measured values of μ_{coil} as a function of the relative distance $2x/l$ of the coil from the mid-point of the rod.

The effective height can now be deduced by taking the value obtained in the first design (0.0033), and multiplying it by the above factors, thus giving:

$$h = 0.0033 \times 1.1 \times 0.63 = 0.0023 \text{ m}.$$

The overall performance then is:

$$h \cdot Q = 0.0023 \times 200 = 0.46.$$

A comparison of this figure with the value of 0.55 found for the first design reveals that the higher quality factor has in fact been obtained at the expense of the overall performance.

A closer investigation of the last design will reveal that some FERROXCUBE material has been wasted, since the end of the rod farthest away from the coil will have very little effect on the pick-up of the aerial. The merit of the design is mainly that it provides a method of trimming the coil and the possibility of placing a further coil for a second wave range at the other end of the rod.

The other solution for attaining the required value of Q is to place the coil at the mid-point of a rod of shorter length. It is seen from fig. 10 that for $1/Q = 50 \times 10^{-4}$ the coil permeability will again be approximately 11. The required length of the rod can now be obtained from fig. 14, which gives the coil permeability μ_{coil} as a function of the ratio l/a for a rod of 8 mm × 200 mm and a coil with $a = 5$ mm.

Fig. 14 shows that the rod will have to be shortened to 100 mm. Since the coil permeability is equal to that in the previous example, the number of turns should also be the same. The rod permeability will, however, be less. It is seen from fig. 5 that μ_{rod} is reduced from 115 to 65, so that the effective height now is:

$$h = 0.0023 \times \frac{65}{115}\,\frac{1}{0.63} = 0.0020 \text{ m}.$$

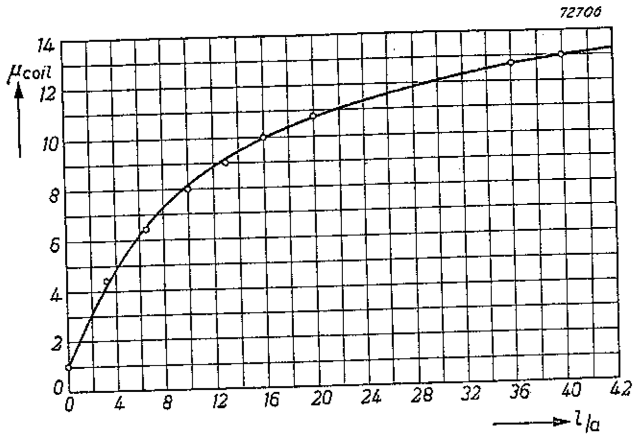

Fig 14. Measured values of μ_{coil} as a function of the ratio l/a.

This is only slightly less than the figure for the former design, but only half the quantity of magnetic material has been used. It may be useful, therefore, to develop from the last design a third, again using the original quantity of FERROXCUBE but reducing the ratio l/d to 12.5.

All dimensions will now have to be increased by a factor $\sqrt[3]{2} = 1.26$, after which the increase in overall performance can be judged from eq. (7).

Since Q is determined mainly by the iron losses, it will remain substantially unchanged, so that only l and $\sqrt{d}$ are affected. This gives the effective height of the third design:

$$h = 0.0020 \times 1.26\,\sqrt{1.26} = 0.0028 \text{ m}.$$

Since the design was based on a quality factor of $Q = 200$, the overall performance now becomes $h \cdot Q = 0.56$.

These examples show clearly that for aerial rods with a high quality factor relatively short and thick cores should be employed. This agrees with measurements carried out on a rod of 14 mm × 100 mm, which gave a quality factor of 250 with a coil of $a = 17$ mm.

It might be expected from fig. 10 or 11 that choosing $\mu_{coil} = 4$, which still gives a quality factor $Q = 200$, would be favourable, because, according to eq. (7), a decrease of μ_{coil} corresponds to an increase of the output voltage E_o. However, figs 13 and 14 show that the position of the coil, respectively the length of the rod, are then very unfavourable for obtaining a high output voltage.

As the quality factor is to all intents and purposes determined by the iron losses, it will decrease with increasing frequency, as can be expected from the trend of the curves of fig. 8. The measured results of two types of aerial rods are tabulated below.

Frequency f	Quality factor Q	
600 kc/s	206	250
1 Mc/s	165	218
1.4 Mc/s	125	173

DIRECTIVITY

If the rod is at an angle Θ with the direction of the magnetic field B of the transmitter, only the component $B \cos \Theta$ in the direction Θ of the rod will be active. Reception will be nil when the rod points towards the transmitter, the polar diagram thus consisting of two circles.

This also applies to loop-aerials, but here interference is experienced from capacitive reception due to the dimensions of the loop. As a results the diagram is usually distorted and becomes more or less cardioidal. Since, however, the dimensions of the coil on a rod are very small, the polar diagram can be expected to consist of two circles, and this is confirmed by the diagram shown in fig. 15 obtained from actual measurements.

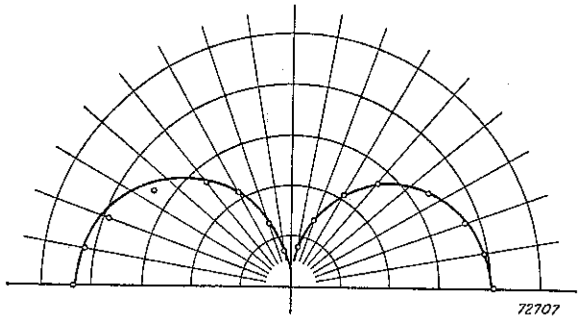

Fig. 15. Polar diagram of an aerial rod.

For "walkie-talkie" or personal portable types of receivers this directivity is a disadvantage, as dead spots will occur every time the set is turned into the zero position. A solution can, however, be found by combining the aerial rod with a small capacitive aerial. On the other hand, where a receiver is operated on a table, the zero position can be a welcome aid to improving selectivity.