Radiation Efficiency of Electrically Small Multiturn Loop Antennas

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Abstract—**The radiation efficiency of electrically small multiturn loop antennas is calculated using an ohmic resistance which includes both the normal skin effect and the additional loss due to the proximity effect.**

I. Introduction

The single-turn loop has been the subject of much investigation, and, from the practical standpoint, adequate design data are available for this structure [1], [2]. The multiturn loop of unrestricted electrical size has received much less attention. The available solutions are for the "one-dimensional" current distribution and are therefore, strictly speaking, only valid for loops with spacings between turns that are large compared with the wire diameter [3], [4].

In practical applications the electrically small loop is often used. The ohmic resistance of small loops is in general much larger than the radiation resistance; thus radiation efficiencies are low and greatly dependent on the ohmic resistance. In an effort to increase the radiation efficiency, multiturn structures are often employed. The radiating properties (radiation resistance and field pattern) of electrically small single- or multiturn loops are easily derived, either directly from the integral form of Maxwell's equations [1], [5] or as a limiting case of one of the more general analyses mentioned previously. These methods are usually concerned with perfectly conducting wires and thus provide no information concerning the ohmic loss of the antenna.

The ohmic resistance of a small loop is usually taken to be the same as that of a straight conductor equal in length to the uncoiled loop. This assumption is adequate for the single-turn loop, but not for the multiturn case. In a multiturn loop the distribution of current over the conductor cross section is determined by two effects—the normal skin effect and a proximity effect. For close conductor spacings, the distribution of current due to the proximity effect can cause an increase in the ohmic resistance which is larger than the skin effect resistance alone. This increase in the ohmic resistance, which is normally unimportant in large antennas, greatly affects calculations of power radiated by electrically small transmitting loops.

II. Resistance of Parallel Wires

The current distribution over the cross section of a system of geometrically parallel wires is determined by a combination of skin and proximity effects. If the skin depth d_s in the conductor is small compared to the wire radius a

$$\frac{a}{d_s} \gg 1, \qquad d_s = \left(\frac{2}{\omega\mu_0\sigma}\right)^{1/2} \tag{1}$$

the current is confined to a thin layer near the conductor surface. In (1) ω is the angular frequency, σ the conductivity of the wire, and μ_0 the free-space permeability. The proximity effect causes a nonuniform distribution of the current in the layer. This is illustrated in Fig. 1 for three closely spaced wires carrying equal currents in the same direction. Subject to (1) the resistance per unit length of a system of n equally spaced geometrically parallel conductors carrying equal currents in the same direction can be calculated

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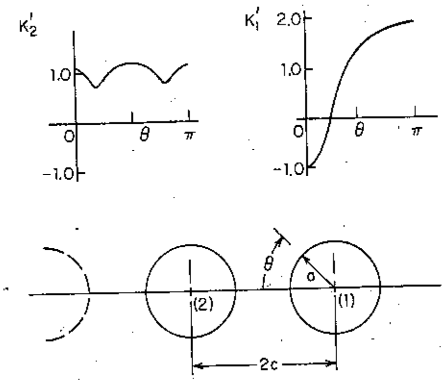

Fig. 1. Normalized surface current distribution $K_m'(\theta) = 2\pi a K_m(\theta)$ on three closely spaced wires ($c/a = 1.10$), each carrying total current of 1 A. Note negative currents on outside cylinders due to proximity effect.

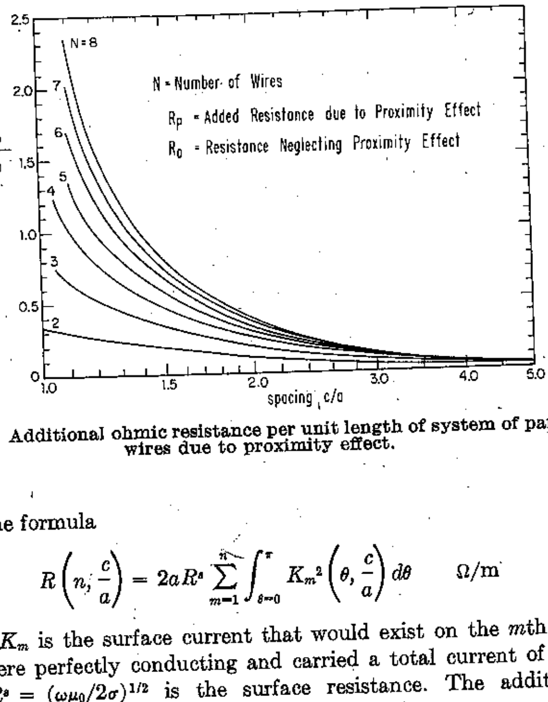

Fig. 2. Additional ohmic resistance per unit length of system of parallel wires due to proximity effect.

from the formula

$$R\left(n, \frac{c}{a}\right) = 2aR^s \sum_{m=1}^{n} \int_{\theta=0}^{\pi} K_m^2\left(\theta, \frac{c}{a}\right) d\theta \qquad \Omega/\text{m} \tag{2}$$

where K_m is the surface current that would exist on the mth wire if it were perfectly conducting and carried a total current of 1 A, and $R^s = (\omega\mu_0/2\sigma)^{1/2}$ is the surface resistance. The additional ohmic resistance due to the proximity effect is then

$$R_p\left(n, \frac{c}{a}\right) = R_0\left(\frac{R(n, c/a)}{R_0} - 1\right) \tag{3}$$

where R_0, the skin effect resistance alone, is given by the Rayleigh formula

$$R_0 = \frac{nR^s}{2\pi a} \qquad \Omega/\text{m}. \tag{4}$$

The results of recent calculations [6] of the quantity R_p/R_0 for various numbers of wires and spacings are presented in Fig. 2. For close spacings the proximity effect can more than double the ohmic resistance of the system of conductors.

III. Radiation Efficiency

The model chosen to represent the electrically small multiturn loop antenna is illustrated in Fig. 3. All turns of the loop are circular and lie in parallel planes. The straight segments of wire interconnecting the turns and the feed wires of the delta-function generator are short, parallel, and closely spaced. These are assumed to have negligible ohmic resistance compared to that of the overall circuit since they represent a small fraction of the total conductor length when $2c \ll b$. The dimension $2c$ is exaggerated in Fig. 3.

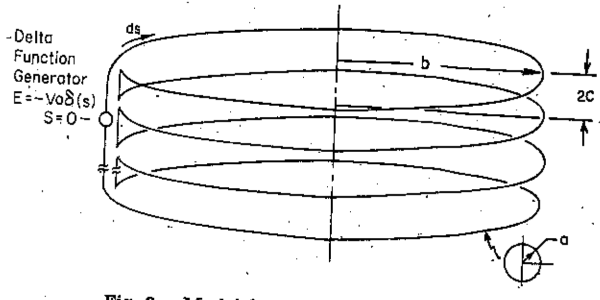

Fig. 3. Model for multiturn loop antenna.

The multiturn loop with n turns will have essentially the same total current I at any conductor cross section provided the total length of the loop wire is much less than the free-space wavelength at the operating frequency, that is,

$$n\beta_0 b \ll 1 \tag{5}$$

where $\beta_0 = 2\pi/\lambda$ is the free-space propagation constant.

The radiation resistance is given by the well-known formula [1]

$$R_{\mathrm{rad}} = 20\pi^2 n^2 \beta_0{}^4 b^4. \tag{6}$$

With the following additional constraint placed on the wire radius a and turn spacing c,

$$n^2c^2 \ll b^2, \qquad c > a \tag{7}$$

the ohmic resistance of the loop is approximately

$$R_{\mathrm{ohmic}} = 2\pi b R\left(n, \frac{c}{a}\right) \tag{8}$$

where $R(n,c/a)$ is the resistance per unit length of a system of parallel wires with the same wire radius and spacing.

The radiation efficiency of the electrically small multiturn loop is [7]

$$E_A = \frac{20\pi^2 n^2 \beta_0{}^4 b^4}{20\pi^2 n^2 \beta_0{}^4 b^4 + \dfrac{nb}{a} R^s \left(\dfrac{R_p}{R_0} + 1\right)}. \tag{9}$$

After rearranging terms, (9) becomes

$$E_A = \frac{1}{1 + \dfrac{8.48 \times 10^{-10} (f_{\mathrm{MHz}}\sigma_r)^{1/2}}{n(b')^3 a'} \left(\dfrac{R_p}{R_0} + 1\right)} \tag{10}$$

where a' and b' are the wire radius and loop radius normalized to the free-space wavelength, f_{MHz} is the frequency in megahertz, and σ_r is the ratio of the conductivity of the loop wire to that of copper. In Fig. 4 the efficiency is plotted as a function of the dimensionless quantity $(b')^3a'/(f_{\mathrm{MHz}}\sigma_r)^{1/2}$ and the number of turns. The dashed lines are for no loss due to the proximity effect ($R_p/R_0 = 0$), while the solid lines include the proximity effect for a spacing $c/a = 1.10$. For most practical applications these two lines will give an upper and lower bound on the efficiency obtainable with various spacings. Intermediate values can be calculated using the results presented in Fig. 2 and (10).

Neglecting the proximity effect can lead to large errors in the calculation of radiation efficiency. For example, in Fig. 4, when the proximity effect is neglected, the calculated efficiency of a three-turn loop can be larger than the actual efficiency of an eight-turn loop of the same size with close conductor spacing ($c/a = 1.10$). Electrically small loop antennas are usually operated with a suitable matching network. The components in the matching network often introduce losses as large as the ohmic loss of the antenna. The overall radiation efficiency of the antenna-matching network combination is then

$$E = E_A \cdot E_m \tag{11}$$

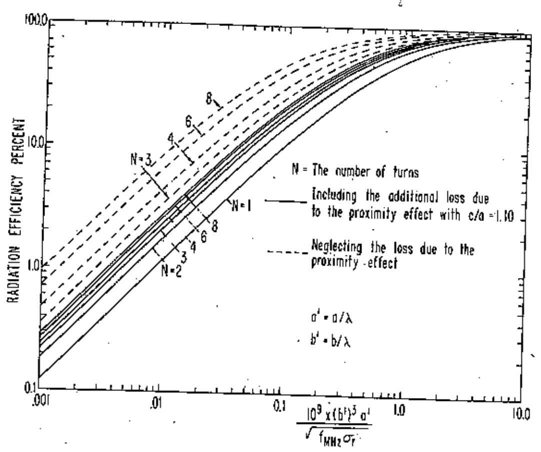

Fig. 4. Radiation efficiency of electrically small multiturn loop antennas.

where E_A and E_m are the efficiency of the antenna and matching network individually. In this communication only E_A is considered; for a discussion of matching network efficiency see Wheeler [8].

IV. Conclusion

Two separate calculations of the radiation efficiency of small multiturn loops have been made. The first includes the added resistance due to the close proximity of turns, and the second neglects all proximity losses, i.e., considers the ohmic resistance of the loop to be the same as that for a straight conductor equal in length to the uncoiled loop. A comparison of the results for these two cases indicates that the proximity effect is an important factor in making accurate calculations of the radiation efficiency, especially for loops whose efficiency is below 10 percent.

Acknowledgment

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Stationary Expressions for Scattering Coefficients of Rectangular Waveguides with Dielectric Plugs Constituting a Finite Planar Array

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***Abstract*—The stationary expressions for the scattering coefficients are given for a finite planar waveguide array (FPWA) with dielectric plugs whose dielectric constants and lengths are arbitrary.**

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