Radio Science, Volume 18, Number 5, pages 664–674, September–October 1983

Loop antennas for directive transmission into a material half space

Glenn S. Smith and Lam N. An[1]

School of Electrical Engineering, Georgia Institute of Technology

(Received January 24, 1983; accepted May 26, 1983.)

The horizontal circular loop and the coaxial array of loops above a material half space are studied as antennas for directive transmission into the half space. In a practical situation the loops might be located in air with the directive transmission into the earth. In determining the optimum geometry for the single loop and the array, the far-zone field patterns and directivities of these antennas when placed over lossless dielectrics are considered first. The directive properties for the lossless dielectric are found to be indicative of those for the same antenna over a medium with low loss when proper account is taken of the exponential attenuation experienced in the lossy medium. Parametric studies are used to obtain the maximum directivities for these antennas. For the single loop of resonant size, the optimum height over the interface is determined, and for the two-element array consisting of a driven loop of resonant size with a single parasite, the optimum size and spacing of the parasitic reflector are found. Measured electric field patterns and gains of model antennas above an interface between air and fresh water are in good agreement with the theoretical results.

INTRODUCTION

In an earlier paper [*An and Smith*, 1982] a comprehensive theoretical analysis with experimental confirmation was presented for the circular-loop antenna near a planar interface separating two semi-infinite material regions. The numerical results presented in that work showed that a loop in free space over a material half space, such as the earth, could have a directive field pattern into the half space when the loop is close to the interface and near resonant size (the circumference of the loop is approximately one wavelength in free space). In this paper the analysis is extended to treat a coaxial array of circular loops above the interface, and numerical results are presented that demonstrate the optimization of the single loop and the two-element array (driven loop with a parasitic reflector) for maximum directivity into the half space.

COAXIAL ARRAY OF CIRCULAR-LOOP ANTENNAS

The coaxial array of n circular-loop antennas ($i = 1, 2, 3, \ldots, n$) over a planar interface is shown in Figure 1. Each of the perfectly conducting loops is driven at the angular position $\phi = 0$ by a delta function generator of voltage V_{0i}. The radius of the loop conductor, the radius of the loop, and the height of the loop above the interface are denoted by a_i, b_i, and h_i, respectively. The spacing between loops i and j is d_{ij}.

For a harmonic time dependence exp $(j\omega t)$ the electrical constitutive parameters for the half space, region 1, containing the loops are the effective permittivity ε_{e1} and the effective conductivity σ_{e1}; the parameters for the other half space, region 2, are ε_{e2} and σ_{e2}. Both materials are assumed to be nonmagnetic, $\mu_1 = \mu_2 = \mu_0$. The complex wave number in either medium is

$$k_i = \beta_i - j\alpha_i = \omega(\mu_0 \xi_i)^{1/2} \qquad \alpha_i \geq 0 \tag{1}$$

where $\xi_i = \varepsilon_{ei} - j\sigma_{ei}/\omega$, and the wave impedance is $\zeta_i = (\mu_0/\xi_i)^{1/2}$.

The current in each of the driven loops can be expressed as a Fourier cosine series:

$$I_i(\phi) = \sum_{m=0}^{\infty} h(m) I_{mi} \cos(m\phi) \tag{2}$$

where

$$\begin{aligned} h(m) &= 1 \qquad m = 0 \\ h(m) &= 2 \qquad m \neq 0 \end{aligned} \tag{3}$$

The coefficients I_{mi} are determined by requiring the tangential component of the electric field, E_ϕ, to sat-

[1] Now at Bell Telephone Laboratories.

Paper number 3S0966.
0048-6604/83/0910-0966$08.00

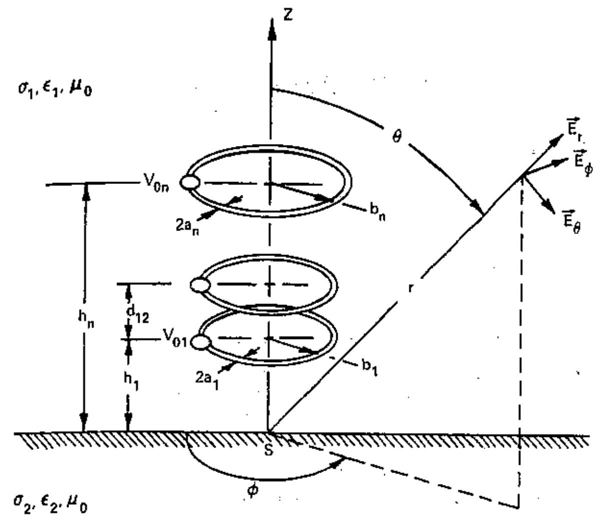

Fig. 1. Coaxial array of circular-loop antennas near a planar interface.

isfy the boundary condition at the surfaces of the perfectly conducting loops:

$$E_{\phi i} = E_{p\phi i} + E_{s\phi i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} E_{\phi ij} = -V_{0i}\,\delta(\phi)/b_i \qquad (4)$$

$$i = 1, 2, 3, \cdots, n$$

The three field components in (4) are the primary field of the isolated loop, which is the field of the ith loop when it is in an infinite medium with the properties of region 1:

$$E_{p\phi i} = \frac{-j\zeta_1}{2b_i} \sum_{m=0}^{\infty} h(m) I_{mi}\, a_{mi} \cos(m\phi) \qquad (5)$$

the secondary field, which is due to the interaction of the ith loop with the half space, region 2:

$$E_{s\phi i} = \frac{-j\zeta_1}{2b_i} \sum_{m=0}^{\infty} h(m) I_{mi}\, b_{mi} \cos(m\phi) \qquad (6)$$

and the field due to the current in the jth loop, which also includes the interaction of the jth loop with the half space:

$$E_{\phi ij} = \frac{-j\zeta_1}{2b_i} \sum_{m=0}^{\infty} h(m) I_{mj}\, c_{mij} \cos(m\phi) \qquad (7)$$

Formulas for the coefficients a_{mi} and b_{mi} in (5) and (6) are given by *King and Smith* [1981] and *An and Smith* [1982], respectively. The coefficients c_{mij} are

$$c_{mij} = G_{pij}(m) + G_{sij}(m) \qquad (8)$$

where G_{pij} and G_{sij} are given by equations (32*c*) and (33*c*) of *An and Smith* [1982] with $\rho = b_i$, $z = -h_i$, $b = b_j$, and $h = h_j$:

$$G_{pij}(m) = -\int_0^{\infty} (k_1/\lambda\gamma_1)[m^2(\gamma_1/k_1)^2 J_m(\lambda b_j) J_m(\lambda b_i) - b_j b_i \lambda^2 J_m'(\lambda b_j) J_m'(\lambda b_i)] e^{-\gamma_1|h_j - h_i|}\, d\lambda \qquad (9a)$$

$$G_{sij}(m) = \int_0^{\infty} (k_1/\lambda\gamma_1)[m^2(\gamma_1/k_1)^2 R_e(\lambda, k_1, k_2) J_m(\lambda b_j) J_m(\lambda b_i) + b_j b_i \lambda^2 R_m(\lambda, k_1, k_2) J_m'(\lambda b_j) J_m'(\lambda b_i)] e^{-\gamma_1(h_j + h_i)}\, d\lambda \qquad (9b)$$

where

$$\gamma_i = (\lambda^2 - k_i^2)^{1/2} \qquad i = 1, 2 \qquad (10)$$

and the reflection coefficients R_e and R_m are

$$R_e(\lambda, k_1, k_2) = (k_{21}^2\gamma_1 - \gamma_2)/(k_{21}^2\gamma_1 + \gamma_2) \qquad (11a)$$

$$R_m(\lambda, k_1, k_2) = (\gamma_1 - \gamma_2)/(\gamma_1 + \gamma_2) \qquad (11b)$$

with $k_{21} = k_2/k_1 = 1/k_{12}$. After inserting (9*a*) and (9*b*) into (8) and some rearrangement,

$$c_{mij} = \int_0^{\infty} \left\{ \frac{m^2\gamma_1}{\lambda k_1} J_m(\lambda b_i) J_m(\lambda b_j) [R_e(\lambda, k_1, k_2) - e^{+2\gamma_1 h_{\min}}] + \frac{\lambda k_1}{\gamma_1} b_i b_j J_m'(\lambda b_i) J_m'(\lambda b_j) [R_m(\lambda, k_1, k_2) + e^{+2\gamma_1 h_{\min}}] \right\} \cdot e^{-\gamma_1(h_i + h_j)}\, d\lambda \qquad (12)$$

where $h_{\min} = \min(h_i, h_j)$. Note that $c_{mij} = c_{mji}$.

When (5), (6), and (7) are inserted in (4) and the delta function expanded as a Fourier cosine series, a set of linear equations results for the coefficients I_{mi}:

$$j\pi\zeta_1 \left[(a_{mi} + b_{mi}) I_{mi} + \sum_{\substack{j=1 \\ j \neq i}}^{n} c_{mij} I_{mj} \right] = V_{0i} \qquad (13)$$

$$i = 1, 2, 3, \cdots, n$$

or in matrix notation,

$$[Y_m]\,[I_m] = [V_0] \qquad (14a)$$

where the elements in the symmetric $n \times n$ admittance matrix $[Y_m]$ are

$$Y_{mij} = j\pi\zeta_1(a_{mi} + b_{mi}) \qquad i = j$$
$$Y_{mij} = j\pi\zeta_1 c_{mij} \qquad i \neq j \qquad (14b)$$

The mth Fourier series coefficients for the currents on all the loops are determined by solving the system of linear equations (14*a*) for the column vector $[I_m]$. For numerical evaluation a finite number of terms are used in the Fourier series (typically 20 terms), so