ANTENNAS

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Also 1988 2nd edn.

NEW YORK TORONTO LONDON

McGRAW-HILL BOOK COMPANY

1950

(6-38) may be applied to (6-34) and (6-35) with an error which is about 1 per cent or less. Equations (6-34) and (6-35) then become

$$E_\phi = \frac{60\pi\beta a[I]\beta a \sin\theta}{2r} = \frac{120\pi^2[I]\sin\theta}{r}\frac{A}{\lambda^2} \tag{6-39}$$

$$H_\theta = \frac{\beta a[I]\beta a \sin\theta}{4r} = \frac{\pi[I]\sin\theta}{r}\frac{A}{\lambda^2} \tag{6-40}$$

These far-field equations for a small loop are identical with those obtained in earlier sections (see Table 6-1).

6-8. Radiation Resistance of Loops.[1] To find the radiation resistance of a loop antenna, the Poynting vector is integrated over a large sphere yielding the total power W radiated. This power is then equated to the square of the effective current on the loop times the radiation resistance R_r.

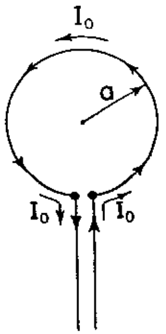

Fig. 6-11. Loop and transmission line.

$$W = \frac{I_0^2}{2}R_r \tag{6-41}$$

where I_0 = peak current in time on the loop. The radiation resistance so obtained is the value which would appear at the loop terminals connected to the transmission line as shown in Fig. 6-11. The situation shown in Fig. 6-11 occurs naturally only on small loops. However, it will be assumed that the current is uniform and in phase for any radius a, this condition being obtained by means of phase shifters, multiple feeds, or other devices (see Sec. 14-20).

The average Poynting vector of a far field is given by

$$P_r = \tfrac{1}{2}|H|^2 \operatorname{Re} Z \tag{6-42}$$

where $|H|$ is the absolute value of the magnetic field and Z is the intrinsic impedance of the medium, which in this case is free space. Substituting the absolute value of H_θ from (6-35) for $|H|$ in (6-42) yields

$$P_r = \frac{15\pi(\beta a I_0)^2}{r^2} J_1^2(\beta a \sin\theta) \tag{6-43}$$

The total power radiated W is the integral of P_r over a large sphere. That is,

$$W = \iint P_r\, ds = 15\pi(\beta a I_0)^2 \int_0^{2\pi}\int_0^{\pi} J_1^2(\beta a\sin\theta)\sin\theta\, d\theta\, d\phi \tag{6-44}$$

[1] The procedure follows that given by Foster, Loop Antennas with Uniform Current, *Proc. I.R.E.*, **32**, 603–607, October, 1944.

or

$$W = 30\pi^2(\beta a I_0)^2 \int_0^{\pi} J_1^2(\beta a\sin\theta)\sin\theta\, d\theta \tag{6-45}$$

In the case of a loop that is small in terms of wavelengths, the approximation of (6-38) can be applied. Thus (6-45) reduces to

$$W = \frac{15}{2}\pi^2(\beta a)^4 I_0^2 \int_0^{\pi} \sin^3\theta\, d\theta = 10\pi^2\beta^4 a^4 I_0^2 \tag{6-46}$$

But the area $A = \pi a^2$ so (6-46) becomes

$$W = 10\beta^4 A^2 I_0^2 \tag{6-47}$$

Assuming no antenna losses, this power equals the power delivered to the loop terminals as given by (6-41). Therefore,

$$R_r\frac{I_0^2}{2} = 10\beta^4 A^2 I_0^2 \tag{6-48}$$

and

$\left[\frac{(2\pi)^4 Z_c}{6\pi}\right] \leftarrow$

$$R_r = 31{,}171\left(\frac{A}{\lambda^2}\right)^2 = 197C_\lambda^4 \quad \text{ohms} \tag{6-49}$$

or

$$R_r \simeq 31{,}200\left(\frac{A}{\lambda^2}\right)^2 \quad \text{ohms} \tag{6-50}$$

This is the radiation resistance of a small single-turn loop antenna, circular or square, with uniform in-phase current. The relation is about 2 per cent in error when the loop perimeter is $\frac{1}{3}$ wavelength. A circular loop of this perimeter has a diameter of about $\frac{1}{10}$ wavelength. Its radiation resistance by (6-50) is nearly 2.5 ohms.

The radiation resistance of a small loop consisting of one or more turns is given by[1]

$$R_r = 31{,}200\left(n\frac{A}{\lambda^2}\right)^2 \quad \text{ohms} \quad = (NA)^2 \frac{Z_c}{6\pi} k_c^4$$

where n = number of turns

Let us now proceed to find the radiation resistance of a circular loop of any radius a. To do this we must integrate (6-45). However, the integral of (6-45) may be reexpressed. Thus, in general,[2]

$$\int_0^{\pi} J_1^2(x\sin\theta)\sin\theta\, d\theta = \frac{1}{x}\int_0^{2x} J_2(y)\, dy \tag{6-51}$$

[1] A. Alford and A. G. Kandoian, Ultrahigh-frequency Loop Antennas, Trans. *A.I.E.E.*, **59**, 843–848, 1940.

[2] G. N. Watson, "A Treatise on the Theory of Bessel Functions," Cambridge University Press, London, 1922.