NO. 472 ALTERNATING-CURRENT RESISTANCE & INDUCTANCE OF SINGLE LAYER COILS
C. N. HICKMAN MAY 3, 1923

Substituting in equation (12) and taking the real parts

$$\frac{R}{R_0}=U_{0/1}+\frac{G}{U_{0/2}+\frac{\lambda^2E^2}{U_{0/2}}}+\frac{K}{U_{1/3}+\frac{\lambda^2F^2}{U_{1/3}}} \tag{13}$$

Taking the imaginary parts and noting that $\frac{\omega}{R_0}=\frac{\lambda}{l}$

$$L=lV_{0/1}-2l\log a-4l\log\left(\frac{N-1}{2}\right)!+2lS-\frac{lG}{\frac{U_{0/2}^2}{E}+\lambda^2E}-\frac{lK}{\frac{U_{1/3}^2}{F}+\lambda^2F} \tag{14}$$

Where L is the self-inductance of the conductor under investigation plus the mutual inductance between it and all the other conductors, the frequency being $\frac{\omega}{2\pi}$.

A more useful formula will be obtained by assuming that the ratio of the inductance expressed by equation (14) to the inductance of the same system for zero frequency is equal to the corresponding ratio of the inductances of a coil, the length of which is great with respect to its diameter.

For zero frequency formula (14) reduces to:

$$L_0=\frac{l}{2}-2l\log a-4l\log\left(\frac{N-1}{2}\right)!+2lS$$

The change in the inductance due to the alternating current is:

$$\Delta L=L_0-L=\frac{l}{2}-lV_{0/1}+\frac{lG}{\frac{U_{0/2}^2}{E}+\lambda^2E}+\frac{lK}{\frac{U_{1/3}^2}{F}+\lambda^2F}$$

The ratio of the alternating-current inductance to the direct-current inductance is

$$\frac{L}{L_0}=\frac{L_0-\Delta L}{L_0}=1-\frac{\Delta L}{L_0}=1-\frac{\frac{1}{2}-V_{0/1}+\frac{G}{\frac{U_{0/2}^2}{E}+\lambda^2E}+\frac{K}{\frac{U_{1/3}^2}{F}+\lambda^2F}}{\frac{1}{2}+2S-2\log a-4\log\left(\frac{N-1}{2}\right)!} \tag{15}$$

Formulas (13) and (15) represent, respectively, the ratios of the alternating-current resistance and inductance to the direct-current resistance and inductance of a system of parallel go and return conductors. These formulas, as explained in the beginning,

will be approximately correct for long solenoids if D represents the radius of the coil.

If the number of conductors of a system of parallel go and return circuits is small with respect to their distance apart, the effect of the return circuit on the distribution of the current in the upper conductors will be negligible.

For a system of parallel conductors without a return these formulas become

$$\frac{R}{R_0}=U_{0/1}+\frac{\pi^4\lambda^2a^4}{54\,s^4\left[U_{1/3}^2+\frac{\lambda^2F^2}{U_{1/3}}\right]} \tag{16}$$

$$\frac{L}{L_0}=1-\frac{\frac{1}{2}-V_{0/1}+\frac{\pi^4\lambda^2a^2}{54\,s^4\left[\frac{U_{1/3}^2}{F}+\lambda^2F\right]}}{\frac{1}{2}-2\log a+2(N-1)\log s-4\log\left(\frac{N-1}{2}\right)!} \tag{17}$$

These formulas will be approximately correct for single-layer coils whose lengths are short compared to their diameters.

Formula (13) gives the ratio of the alternating-current resistance to the direct-current resistance for a single-layer coil, the length of which is great with respect to its diameter.

Formula (15) gives the corresponding ratio for the inductance of the coil.

Formula (16) gives the ratio of the alternating-current resistance to the direct-current resistance of a single-layer coil, the length of which is small with respect to its diameter.

Formula (17) gives the corresponding ratio for the inductance of the coil.

The following notation is used in the above formulas:

- R = Alternating-current resistance.
- R_0 = Direct-current resistance.
- L = Alternating-current inductance.
- L_0 = Direct-current inductance.
- N = Total number of conductors in a plane or the number of turns on the coil.
- a = Radius of conductor.
- D = Distance between floors or the radius of the coil.
- s = Spacing between centers of adjacent conductors.
- $\lambda = \frac{\pi\,\omega\,a^2}{\sigma}$
- ω = 2π times the frequency.
- σ = Resistivity in electromagnetic cgs units.