Effective Resistance of Inductance Coils at Radio Frequency.—Part I.

By S. Butterworth. [R144

(*Admiralty Research Laboratory, Teddington.*)

1. Introduction.

OUR main object in the following work is to develop formulæ suitable for the determination of the effective resistance of inductance coils at radio frequencies, and from these formulæ to obtain a scheme of design which aims at making the resistance as small as possible.

That there is need for such a scheme of design is clearly shown by quoting the constants of two coils, both constructed at the National Physical Laboratory, and both intended for use at a frequency of one million cycles per second. The first coil is one used in the Standard Harmonic Wavemeter[1] at the National Physical Laboratory, and is in the form of a flat spiral nearly 20 cms. in diameter, wound with copper strip. Its inductance is 45 microhenries and its resistance at a wavelength of 300 metres is 0.89 ohm, that is, 19.8 ohms per millihenry.

The second coil is that described by Mr. Wilmotte in the May number of *E.W. & W.E.* It is a square coil of side 7.5 cms. and winding length 4.5 cms. The wire used is No. 22 S.W.G. Its calculated inductance is 77 microhenries and its resistance at 300 metres is 1.58 ohms; that is, 20.5 ohms per millihenry. The point we wish to emphasise in regard to these two coils is that, although the former coil is much larger than the latter, and was decided upon after considerable experimental work, yet its efficiency as expressed in ohms per millihenry is practically the same as the much more modest coil of Mr. Wilmotte, which may be wound by any beginner at the cost of a few pence.

As regards the factors contributing to the high-frequency resistance of inductance coils, great stress has been laid recently upon the importance of dielectric losses. Now, although these losses may play a very important part in very large transmitting coils, or even in receiving coils when used at very short wave-lengths, it is easily possible, by taking very ordinary precautions, to reduce these losses to a small fraction of the whole in receiving coils intended for the broadcasting range of wavelengths. The present writer has wound coils with D.C.C. wire upon solid wooden formers, and on comparing the calculated copper resistance with the total measured resistance has found the ratio of copper to total loss to lie between 60 and 70 per cent. at a wavelength of 300 metres, while with coils for which the insulating material is carefully chosen the copper loss has been 80 to 90 per cent. of the whole. Clearly, therefore, any numerical scheme of design must aim at making the *copper* losses a minimum.

It will be shown that the copper losses in inductance coils in which the turns are well spaced divide themselves into two independent portions, one of which diminishes and the other increases as the diameter of the wire is increased. There is therefore a certain diameter of wire for which the two losses "balance"; that is, the rate of diminution with increase of diameter of the one loss is equal to the rate of increase of the other loss; so that at this diameter the losses are at a minimum.

The scheme of design, therefore, is to find for all reasonable shapes of coil the value of this best wire diameter and the corresponding loss. By comparing the minimum losses for each shape of coil we can then arrive at the best shape of coil to suit any prescribed conditions. Since the inductance required fixes the number of turns, it only remains to fix the mode of grouping of the turns in the winding section and the order in which the current shall pass through the turns.

The mode of grouping will only affect the copper losses slightly, and (except in so far as self-capacity modifies the effective resistance) the order of passage of the current through the turns has no influence upon the copper losses, so that these latter points

[1] See the paper entitled "A Self-contained Standard Harmonic Wavemeter." Dye. *Phil. Trans.*, *A*, Vol. 224, p. 259.

are determined by the mechanical means of winding and by the desirability of having low self-capacity. The correct mode of procedure will be illustrated by examples to be given later.

2. Copper Losses in Inductance Coils.—Preliminary Survey.

In Fig. 1 is drawn the section of a circular inductance coil of rectangular winding section, the turns being represented by small circles, each occupying its own small rectangular space. When current flows in the coil, a magnetic field is set up, the lines of force of which are approximately represented by the broken lines in the figure.

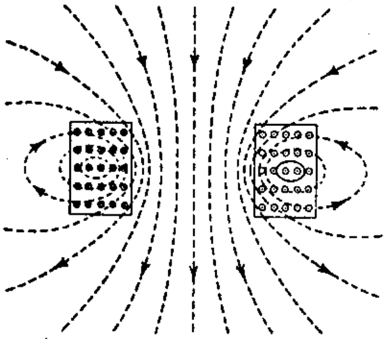

Fig. 1. *Cross section of circular inductance coil of rectangular winding section. The small circles represent the wires composing the turns of the coil and the broken lines are the magnetic lines of force produced by the current in the coil.*

When the current is alternating there are two distinct copper losses in the coil. The first loss occurs whether the current is alternating or not and, for well-spaced turns, is independent of the coiling. It is, in fact, merely the loss due to forcing the current against the resistance of the wire. This loss, however, depends upon the frequency of alternation of the current, increasing as the frequency increases. Since this loss is the same as when the wire is straight we may refer to it as the straight-wire loss, and denote the resistance representing it by R_s. For direct currents R_s, of course, reduces to the ordinary resistance R.

The second loss is due to the alternations of the general magnetic field of the coil. It is a fundamental principle of electro-magnetic induction that when the lines of force of a varying magnetic field thread through any conducting mass there is induced in that mass a circulating or eddy current. This eddy current requires energy for its maintenance, which must be supplied to the mass from the current producing the varying field, and thus the effective resistance of the coil carrying this current is increased. Applying this to the case of the inductance coil shown in Fig. 1, the variation of the general magnetic field of the coil induces eddy currents in each wire, and the energy required increases the effective resistance. Since the added resistance is due to the magnetic field of the coil we will denote it by R_h. We may look upon this resistance as being due to the presence of neighbouring wires, and from this point of view the added resistance is sometimes referred to as the "proximity" resistance.

It will readily be seen that the induced eddy currents do not alter the total current flowing over the whole cross-section of the wire, as they both go and return in the same section. They act rather in *distorting* the distribution of current throughout the section. It is well to consider this distortion more closely, as it will help us when we are dealing with the resistances in a quantitative manner.

The general principle governing the direction of induced currents is the well-known Law of Lenz, namely, that the induced effect always opposes the inducing cause. Since in the present case the inducing cause is the alternation of the magnetic field, the induced current will oppose the action most effectively if, by its own field, it annuls the magnetic field threading through the substance of the wire. The direction of the eddy current must therefore be such that the magnetic field it sets up opposes the inducing magnetic field inside the wire.

Thus, let the circle in Fig. 2 represent one of the right-hand upper wires of the coil of Fig. 1. At the moment when the current is flowing upwards from the paper the field within the wire and due to the current in the wire forms a series of circular counter-clockwise lines of force concentric with the circumference of the wire, while the general field due to the current in the remainder of the coil may be regarded as uniform over the section of the wire and directed up the page.

First, if the general field is absent, a radial wedge, such as AO, imagined cut in the wire, will have acting upon it a downward field. Therefore the eddy current in this wedge must produce an opposing upward field. To do this it must flow towards the observer at A and away from the observer at O. A similar eddy current is induced in every wedge such as AO, and it is immediately seen that the eddy current tends to weaken the main current in the central regions and to strengthen it in the outer regions of the wire. Thus the current tends to concentrate on the outer skin of the conductor, and as the virtual section of current flow is thus diminished the resistance of the wire is increased.

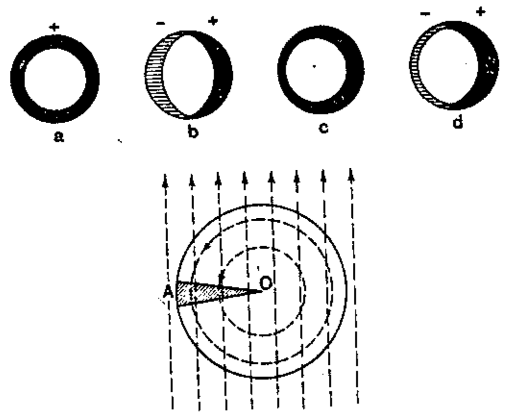

Fig. 2. *The full circle in the main figure represents a wire in the top right-hand half of the coil of Fig.* 1. *The broken circles in the main figure are the lines of force due to the current in this wire and the broken vertical lines are the lines of force due to the remaining wires.* AO *is a wedge imagined cut in the wire. The eddy currents induced in this wedge are considered in the text*: (a) *shows the current distribution in a solitary wire,* (b) *the current induced by a uniform field, and* (c) *and* (d) *the distorted distribution due to* (a) *and* (b) *combined,* (c) *holding when* (a) *predominates and* (d) *when* (b) *predominates.*

Next, if we suppose for the moment that the current in the wire is absent, the general field must produce an eddy current which by the Law of Lenz flows into the paper in the left half and out in the right half of the section of the wire. There are in this case two crescent-shaped sections of current, the left one negative and the right one positive. Superposing the main current, the distortion is seen to consist of a motion of the mean centre of the current in the wire towards the right and a general concentration of the current towards the surface. If the same reasoning be applied to every turn in the coil it is found that the current is, so to speak, trying to get to the nearest boundary of the winding section of the coil, the effort being more vigorous in the case of the inner turns because of the stronger general field there. The phenomenon is, in fact, an attempt to reproduce the "skin" effect for the whole winding section which has been seen to occur in the case of the individual wire.

We may also readily deduce that the two types of energy loss act independently, for while the main current, even when modified by the eddy currents due to the circular field has the same sign in both left and right halves of the wire section, the eddy current induced by the general field is of changed sign in the two halves. Thus if I_m and I_h be the two types of current in a filament of the wire in the right-hand half, the total current in this filament is $I_m + I_h$, while in a symmetrically disposed filament in the left half the total current is $I_m - I_h$. Now the energy loss is proportional to the square of the current, so that for the pair of filaments the energy loss is proportional to $(I_m + I_h)^2 + (I_m - I_h)^2$, that is to $I_m^2 + I_h^2$. Adding, for all filaments, the total energy loss is proportional to $\Sigma I_m^2 + \Sigma I_h^2$.

But this is the result we should have obtained if we had calculated each loss as if the other were absent and *then* added.

As the latter method will enormously simplify our problem it will be the one adopted in the following pages.

The problem of determining the copper resistance in inductance coils is thus capable of analysis into three portions, viz. :—

(*a*) The variation of resistance of a solitary wire with frequency.

(*b*) The energy losses in a wire when placed in an alternating magnetic field.

(*c*) The distribution of magnetic field throughout the winding section of various types of inductance coils.

These problems will be dealt with in turn in the following sections.

3. The Alternating Current Resistance of a Long Straight Wire.

Formulæ for the computation of the alternating current resistance of a long straight wire were supplied nearly forty years ago by Kelvin,[2] Rayleigh[3] and Heaviside.[4] The formulæ as given by them were not in such a form as to be easily used by the electrical engineer, so that, as the use of high frequency currents increased, various writers tried to simplify the mode of presentation. Tables of values of the alternating current resistance of wires of various diameters have been computed by Zenneck[5]; and Rosa and Grover[6] have given tables whereby the A.C. resistance may be found with great ease. The column headed $1+F$ in Table I. has been extracted from the latter set of Tables. Before giving the general solution we will consider the cases of low and high frequencies respectively.

3.1. *Case of Low Frequency.*

We have seen in Section 2 that the effect of the circular alternating field within the substance of a straight wire carrying alternating current is to produce an eddy current which flows in the same direction as the main current in the outer regions of the wire and returns *via* the central regions. The total energy loss in the wire is obtained by adding the energy loss due to this eddy current to the energy loss due to the main current. If the maximum value of the main current be I and the D.C. resistance be R the dissipation of power by the main current is

$$W_1 = \tfrac{1}{2} RI^2 \quad .. \quad .. \quad (1)$$

Now, as regards the eddy current, if E be the average value throughout the wire of the E.M.F. inducing this current and R' be the average value of the resistance of the eddy current path, the dissipation of power by the eddy current is

$$W_2 = \tfrac{1}{2} E^2/R' \quad .. \quad .. \quad (2)$$

as at low frequencies the inductance of the eddy current path may be neglected.

To estimate the value of E, let a be the radius of the wire and let the frequency be $\omega/2\pi$. The field intensity throughout the wedge AO varies uniformly from zero to $2I/a$ as we pass from O to A so that if the wire is of length l the total flux threading through the wedge is $la \times I/a = lI$, and the E.M.F. that acts on the filament of current path which bounds the wedge is ωlI. The mean E.M.F. must be less than this, so that as we are only seeking an approximate solution we will assume that the average E.M.F. is $\tfrac{1}{2}\omega lI$.

The resistance R' of the eddy current path may also be approximated to by imagining the wire to be divided into a central rod and outer tube each of the same resistance, viz., $2R$. If we suppose the eddy current to flow by the outer tube and return by the central rod the resistance R' is clearly $4R$ and then by (2) $W_2 = \omega^2 l^2 I^2/32R$.

It would be surprising if this were the correct result, as we have made two approximations. The equation is, however, correct in form and in actual fact will be correct in magnitude if we replace the number 32 by 24. Hence we write[7]

$$W_2 = \omega^2 l^2 I^2/24R \quad .. \quad (3)$$

Adding W_2 to W_1 the power loss is

$$W_s = W_1 + W_2 = \tfrac{1}{2} RI^2 \left(1 + \frac{\omega^2 l^2}{12R^2}\right) .. \quad (4)$$

and therefore the A.C. resistance is

$$R_s = R\left(1 + \frac{\omega^2 l^2}{12R^2}\right) = R(1+F) \text{ say} \quad .. \quad (5)$$

3.2. Formula (4) will not carry us far up the scale of frequency, as when the term F is beginning to be appreciable the assumptions made in establishing (5) are no longer valid. The formula is, however, useful as it indicates the *form* which the final solution will take.

In regard to the correction F it is important to notice that it is a *pure number*,[8] as it has to be added to the pure number unity. It should also be noted that R in the expression for F must be in c.g.s. units. Since on this system of units an inductance is measured in centimetres, we may for the moment regard the length l as representing some inductance and then we see that F is of the nature of the square of the ratio of a reactance to a resistance, thus verifying its nature as a pure number.

Although the length of the wire appears

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[2] *Math. and Phys. Papers*, Vol. 3, 1889.
[3] *Phil. Mag.*, Vol. 21, 1886.
[4] *Electrical Papers*, Vol. 2, p. 64.
[5] Zenneck, *Wireless Telegraphy*, Table 7; Hoyle, *Standard Tables and Equations in Radio Telegraphy*, Table 18, p. 51.
[6] *Bulletin Bureau of Standards*, p. 226, 1912.

[7] All units are assumed to be electro-magnetic c.g.s. units and all alternating quantities are supposed represented by their maximum values.

[8] *I.e.*, *not* a length, a resistance, or any magnitude of that kind.—Ed.

in the expression for F, yet F is independent of this length, as R is also proportional to the length of the wire.

On expressing R in terms of the resistivity ρ and diameter d of the wire we get

$$F = \frac{\pi^2\omega^2 d^4}{192\rho^2} = \frac{z^4}{192} \text{ say} \quad .. \quad (6)$$

in which, for brevity, we have written

$$d\sqrt{\frac{\pi\omega}{\rho}} = \pi d\sqrt{\frac{2f}{\rho}} = z \quad .. \quad (7)$$

This form is chosen because when the simple form (6) no longer holds for F, F may still be expressed in simple series involving z and z only. To see that this is so we need only know that F always remains a pure number which depends only on d, f and ρ. Now, the only way in which we can associate d, f and ρ so as to yield a pure number is to combine expressions such as $A(d\sqrt{f/\rho})^n$ where A and n are mere numbers. As z is equal to $\pi d\sqrt{2f/\rho}$, we may therefore assert that F is a function of z and of z only.

This law is very important, as by its means we can, from measured values of the A.C. resistance of a particular wire at a series of frequencies, deduce the values of the A.C. resistances of wires of other diameters and of other materials. This is effected by plotting the ratio of A.C. resistance to D.C. resistance of the particular wire against the single variable z.

TABLE I.

Values of the Functions F and G.

d=diameter of wire (cm.); ρ=resistivity (c.g.s. units); f=frequency (cycles per sec.).
$z=\pi d\sqrt{2f/\rho}$. For copper of resistivity 1,700 c.g.s. units $d\sqrt{f}=9.28\,z$, or $z=0.1078\,d\sqrt{f}$.

z	$1+F$	G	z	$1+F$	G	z	$1+F$	G	z	$1+F$	G
0.0	1.000		2.5	1.175	0.2949	5.0	2.043	0.755	10.0	3.799	1.641
0.1	1.000		2.6	1.261	0.3184	5.2	2.114	0.790	11.0	4.151	1.818
0.2	1.000		2.7	1.228	0.3412	5.4	2.184	0.826	12.0	4.504	1.995
0.3	1.000	$z^4/64$	2.8	1.256	0.3632	5.6	2.254	0.861	13.0	4.856	2.171
0.4	1.000		2.9	1.286	0.3844	5.8	2.324	0.896	14.0	5.209	2.348
0.5	1.000	0.00097	3.0	1.318	0.4049	6.0	2.394	0.932	15.0	5.562	2.525
0.6	1.001	0.00202	3.1	1.351	0.4247	6.2	2.463	0.967	16.0	5.915	2.702
0.7	1.001	0.00373	3.2	1.385	0.4439	6.4	2.533	1.003	17.0	6.268	2.879
0.8	1.002	0.00632	3.3	1.420	0.4626	6.6	2.603	1.038	18.0	6.621	3.056
0.9	1.003	0.01006	3.4	1.456	0.4807	6.8	2.673	1.073	19.0	6.974	3.233
1.0	1.005	0.01519	3.5	1.492	0.4987	7.0	2.743	1.109	20.0	7.328	3.409
1.1	1.008	0.02196	3.6	1.529	0.5160	7.2	2.813	1.144	21.0	7.681	3.586
1.2	1.011	0.03059	3.7	1.566	0.5333	7.4	2.884	1.180	22.0	8.034	3.763
1.3	1.015	0.04127	3.8	1.603	0.5503	7.6	2.954	1.216	23.0	8.388	3.940
1.4	1.020	0.0541	3.9	1.640	0.5673	7.8	3.024	1.251	24.0	8.741	4.117
1.5	1.026	0.0691	4.0	1.678	0.5842	8.0	3.094	1.287	25.0	9.094	4.294
1.6	1.033	0.0863	4.1	1.715	0.601	8.2	3.165	1.322	30.0	10.86	5.177
1.7	1.042	0.1055	4.2	1.752	0.618	8.4	3.235	1.357	40.0	14.40	6.946
1.8	1.052	0.1265	4.3	1.789	0.635	8.6	3.306	1.393	50.0	17.93	8.713
1.9	1.064	0.1489	4.4	1.826	0.652	8.8	3.376	1.428	60.0	21.46	10.48
2.0	1.078	0.1724	4.5	1.863	0.669	9.0	3.446	1.464	70.0	25.00	12.25
2.1	1.094	0.1967	4.6	1.899	0.686	9.2	3.517	1.499	80.0	28.54	14.02
2.2	1.111	0.2214	4.7	1.935	0.703	9.4	3.587	1.534	90.0	32.07	15.78
2.3	1.131	0.2462	4.8	1.971	0.720	9.6	3.658	1.570	100.0	35.61	17.55
2.4	1.152	0.2708	4.9	2.007	0.738	9.8	3.728	1.605			
2.5	1.175	0.2949	5.0	2.043	0.755	10.0	3.799	1.641	Large	$(\sqrt{2}z+1)/4$	$(\sqrt{2}z-1)/8$

3.3. *Case of High Frequency.*

The form assumed by F at very high frequencies may be deduced from the knowledge that it is always a function of z and from the fact that the current is then concentrated upon the outer skin of the conductor. If the depth of penetration of the current is small compared with the diameter of the wire, this depth will be practically independent of the diameter of the wire. Under these circumstances the cross section of the path of the high frequency current is proportional to the circumference of the wire, that, is, to its diameter. Hence the high frequency resistance is proportional to $1/d$ or the ratio of the A.C. to D.C. resistance is proportional to d.

But the ratio is also a function of z, and by (7) these two properties can only be reconciled if the ratio is proportional to z. Thus we deduce that at high frequencies $1+F$ varies as the square root of the frequency and inversely as the square root of the resistivity. The numerical multiplier can only be found by recourse to mathematical analysis, and it is found that

$$1+F=\frac{\sqrt{2}}{4}z=0.354z \quad .. \quad (8)$$

We might go a step farther and assert that when z is not extremely large the above estimate is somewhat low, as the effect of the curvature of the wire must be to restrict the depth of penetration to some extent. In fact to a nearer approximation

$$1+F=\frac{(\sqrt{2}\,z+1)}{4} \quad .. \quad (9)$$

3.4 *Case of Moderate Frequencies.*

For intermediate frequencies the formulæ are complicated, and tables are necessary. The values of $1+F$ are given in Table I. It will be seen from the table that the simple formula (6) is good enough for most purposes up to $z=2$, and the simple formula (9) when z exceeds 5.

3.5. *Experimental Verification.*

An experimental verification of the law of increase of resistance of a straight wire with frequency has been made by Kennelly, Laws and Pierce.[9] These experimenters used a copper conductor 1.168 cms. diameter and a range of frequency from 60 to 5,000 cycles per second, so that the value of z ranges from zero to 9. Using a value of resistivity as deduced from the D.C. resistance, the following comparison Table holds.

Frequency		60	306	888	1600	2040	3065	3950	5000
R_s/R	Obsd.	1.005	1.108	1.560	2.045	2.27	2.71	3.03	3.37
	Calcd.	1.004	1.111	1.587	2.042	2.28	2.69	3.03	3.36

4. Eddy Current Losses in a Cylinder when Placed in a Uniform Alternating Magnetic Field.

The second type of copper loss in an inductance coil is that due to the general field of the coil. A glance at Fig. 1 shows that for coils in which the turns are well spaced this loss may be determined if we can obtain an expression for the loss in a wire when placed in a uniform alternating magnetic field the direction of which is perpendicular to the axis of the wire. For coils in which a wavy type of winding is employed the direction of the field is no longer at right angles to the axis of the wire. The loss in a cylinder inclined at any angle to a uniform field is capable of solution, but will not be dealt with here as there is no indication that a wavy winding possesses any electrical advantage over a straight winding.

Solutions of the problem of a cylinder in a transverse field which is alternating slowly have been given by many writers. We need only refer the reader to Howe[10] who gives the proof as a preliminary to a treatment of stranded conductors. At the other end of the scale of frequency the simplest form of solution of the problem met with by the writer is that due to Fortescue[11] A more general treatment covering the whole range of frequency and including

[9] *Trans. Amer. I.E.E.*, Vol. 35, Part 2, 1953, 1915.
[10] *Proc. Roy. Soc., A*, Vol. 93, p. 468, 1917.
[11] *Journ. I.E.E.*, Vol. 61, p. 933, 1923.

the case where the field is not uniform over the section of the cylinder is to be found in a paper by the present writer.[12]

We will first establish the *form* of the expression for the eddy current loss and then deal in detail with the cases of low and of high frequency.

4.1. *Form of Equation for Eddy Current Losses.*

Let H be the strength of the field and let d be the diameter of the cylinder. Also let R be the D.C. resistance of the cylinder. The magnitude of the eddy current induced by the alternations of the field must be proportional to H, so that, since the energy loss is proportional to the square of the eddy current, it must be proportional to H^2. Now the "dimensions" of an energy loss are those of a resistance multiplied by the square of a current, while the "dimensions" of a field intensity are those of a current divided by a length (*e.g.*, the field intensity at distance r from a long straight wire carrying current I is $2I/r$). In order therefore to get the correct "dimensions" we multiply H^2 by d^2 to make it equivalent to the square of a current and then by R to make it equivalent to a rate of energy loss. The true energy loss must then be equal to Rd^2H^2 multiplied by some *numerical* quantity. The total energy loss in the wire is obviously proportional to its length, but this is provided for as the factor R is proportional to the length of the wire. Hence the numerical quantity sought must be independent of the length and can only depend upon the frequency f, the resistivity ρ and the diameter d of the cylinder. But the problem of determining the most general *numerical* quantity which depends only on the above physical quantities has already been dealt with in Section 3, and we conclude that the required numerical quantity is some function of the variable z defined in equation (7). We therefore write for the rate of energy loss due to the transverse field H,

$$W_h = Rd^2H^2G/8 \qquad \therefore \quad (10)$$

where G is a function of z and the divisor 8 is merely introduced to simplify certain later equations.

4.2. *Value of G for Low Frequencies.*

If the frequency is low the magnitudes of the eddy currents are small and we may neglect the field they set up in comparison with the inducing field. This amounts to neglecting the inductance of the eddy current paths, so that the magnitude of the eddy currents may be determined using Ohm's Law.

We first determine the eddy current losses in a thin strip placed athwart the field H and then by supposing the cylinder to be built up of such strips determine by addition the eddy losses in the whole cylinder.

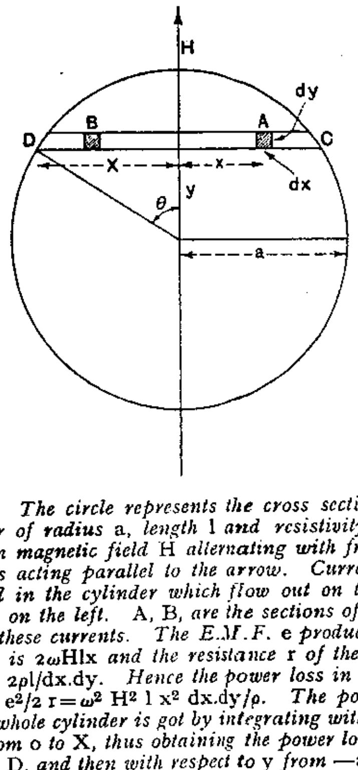

Fig. 3. *The circle represents the cross section of a cylinder of radius* a, *length* l *and resistivity* ρ. *A uniform magnetic field* H *alternating with frequency* ω/2π *is acting parallel to the arrow. Currents are induced in the cylinder which flow out on the right and in on the left.* A, B, *are the sections of path of one of these currents. The E.M.F.* e *producing this current is* 2ωHlx *and the resistance* r *of the current path is* 2ρl/dx.dy. *Hence the power loss in the path* A B *is* $e^2/2r = \omega^2 H^2 l x^2 dx.dy/\rho$. *The power loss in the whole cylinder is got by integrating with respect to* x *from* 0 *to* X, *thus obtaining the power loss in the strip* C D, *and then with respect to* y *from* —a *to* +a. *The latter integration is effected by making the substitutions* X = a sin θ, y = a cos θ *and then integrating with respect to* θ *from* 0 *to* π. *The result is*

$$W_h = \pi\omega^2 H^2 l a^4/8\rho.$$

Referring to Fig. 3 with its explanatory text we see that for low frequencies

$$W_h = \pi\omega^2H^2la^4/8\rho = \pi\omega^2H^2ld^4/128\rho \quad .. \quad (11)$$

But $$Rd^2H^2/8 = \rho lH^2/2\pi,$$

so that by (10)

$$G = \pi^2\omega^2d^4/64\rho^2 = z^4/64 \quad .. \quad (12)$$

This simple form will hold for values of z up to about unity.

[12] *Phil. Trans, A*, Vol. 222, p. 57, 1921.

The equation is more useful than the corresponding one for F, as many cases arise in which, because of the large value of H, the eddy losses are of importance even when G is small. From equation (11) we see that the eddy losses vary directly as the fourth power of the diameter and inversely as the resistivity. As regards the change with diameter the loss is varying in an opposite way to the loss due to a current flowing in the wire. It thus appears that in an inductance coil any attempt to reduce the copper losses by employing wire of the maximum possible diameter may be defeated by the very large loss due to the general field.

As regards the change with resistivity, it is seen that if it is necessary to use metal rods in the neighbourhood of an inductance coil these rods should preferably be of *high* resistivity when the frequency is low. The term "low" frequency requires careful definition. The relevant quantity is the factor z, which involves the wire diameter and its resistivity as well as the frequency. When the term "low" frequency is employed with reference to wires or cylinders it will be taken to mean that z is less than unity.

4.3. *Value of G for High Frequencies.*

The form of G at extremely high frequencies may be deduced from the principle that if the eddy currents are sufficiently powerful they will practically annul the field within the substance of the wire. In the extreme case the distribution of the eddy current is as shown in Fig. 2(*b*) with the two crescent-shaped sections of current path concentrated closely upon the outer surface of the wire. The combined effect of the inducing field and that produced by the eddy currents is that the lines of force curve round instead of passing through the cylinder. In fact, the distribution of the lines of force at extreme frequencies are exactly analogous to the lines of flow of an ideal fluid past a cylindrical obstacle, and mathematically the problem may be dealt with by an analogous system of equations. It is important to notice that this distortion of the field increases the field intensity along the diameter of cross section which is perpendicular to the field and weakens it in directions parallel to the field. These facts will be made use of later in connection with single layer coils.

The depth of penetration of the eddy currents is practically independent of the diameter of the wire, but the dimension of the crescent-shaped path parallel to the direction of the field is equal to the diameter. Hence the cross section of the path is proportional to the diameter. Now the energy dissipated by the eddy current is inversely proportional to the resistance of its path, so that it is proportional to the cross section of the path or to the diameter of the wire. Since in (10) Rd^2 is independent of the diameter it follows that G is proportional to the diameter of the wire. This can only be so if G is merely a multiple of z. The complete theory shows that

$$G = \sqrt{2}\,z/8 \quad \ldots \quad \ldots \quad (13)$$

For finite values of Z this is only an approximation, being the first term of a series. If we include the next term we have

$$G = (\sqrt{2}\,z - 1)/8 \quad \ldots \quad (14)$$

This formula is a good approximation for values of z greater than 5.

From (13) we see that the eddy losses still increase with diameter of wire but the rate of increase is not so rapid as at low frequencies, being proportional to the first instead of the fourth power of the diameter.

From (10) and (13) we see that the eddy loss varies directly as the square root of the resistivity, so that at *high* frequencies improvement is obtained as regards losses in neighbouring metal rods by reducing the resistivity. This is opposite to the law at low frequencies. Therefore as the resistivity is increased there is for any particular rod a resistivity giving maximum eddy losses and any departure from this resistivity will reduce the losses.

When the value of z is such that neither of the simple formulæ (12) or (14) hold, a much more complicated mathematical analysis is necessary, and the values of G given in Table I. must be used.

(To be continued.)

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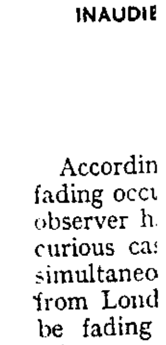

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Effective Resistance of Inductance Coils at Radio Frequency.—Part II.

[R144

By S. Butterworth.

(Admiralty Research Laboratory, Teddington.)

5. Alternating Current Resistance of Two Parallel Wires.

This is the simplest case in which the theory may be applied. If the current is I and the axes of the wires are separated by a distance c, then each wire is situated in a field of intensity $2I/c$ due to the current in the other wire. The eddy loss due to this field is obtained by replacing H by $2I/c$ in (10) and we then have for both wires

$$W_h = RI^2Gd^2/c^2 \quad .. \quad (15)$$

so that the increment in resistance due to this loss is

$$R_h = RGd^2/c^2 \quad .. \quad (16)$$

Adding this to the A.C. resistance R_s of a solitary wire the total resistance is

$$R_c = R_s + R_h = R(1+F+Gd^2/c^2) \quad .. \quad (17)$$

This formula is only accurate if d/c is not too large.

When the wires are close together so that d/c is nearly unity we can no longer neglect the fact that the induced eddy currents distort the field in the plane containing the axes of the two wires. In fact the two terms of (17) are the first two terms of an infinite series. The next term has been calculated and it has been found that (17) must be replaced by

$$R_c = R(1+F+Gd^2/c^2+GHd^4/c^4+\ldots) \quad (18)$$

in which H is a new function of z having the following values:—

z ..	0	1	2	3	4	5	large	
H	0.042	0.053	0.169	0.348	0.466	0.530	0.750	Go and Return
	0.042	0.033	—0.056	—0.152	—0.176	—0.185	—0.250	Currents in same direction

If we assume that the remaining terms are in geometrical progression and therefore write

$$R_c = R\left(1+F+\frac{Gd^2/c^2}{1-Hd^2/c^2}\right) \quad \ldots \quad (19)$$

we obtain a formula which is in close agreement with experimental results for values of d/c up to 0.9.

6. Single-Layer Systems of Spaced Parallel Wires. Short Coils.

Formula (17) will clearly also hold for a two-turn coil provided that c is small compared with the radius of the coil and that d/c is not too large.

If we take three coplanar parallel wires of equal axial separation c, the field acting on each of the outer wires due to the current in the remaining two is $3I/c$ and there is no field acting on the middle wire.

Thus the mean square field for all three wires is

$$(3^2+0+3^2)I^2/3c^2 = 6I^2/c^2$$

Using this in (10) we obtain for the mean value of R_h

$$R_h = 1.5\, RG\, d^2/c^2$$

where R is the D.C. resistance of one wire. For all three wires in series we simply multiply by 3, or, what is the same thing, regard R as the D.C. resistance of the whole system. Thus for a system of three wires the A.C. resistance formula is

$$R_c = R(1+F+1.5Gd^2/c^2) \quad .. \quad (20)$$

The same procedure may be applied to any number of coplanar, equally spaced, parallel wires and in general we get

$$R_c = R(1 + F + uGd^2/c^2) \quad .. \quad (21)$$

in which u depends upon the number of wires in the system. Thus we have

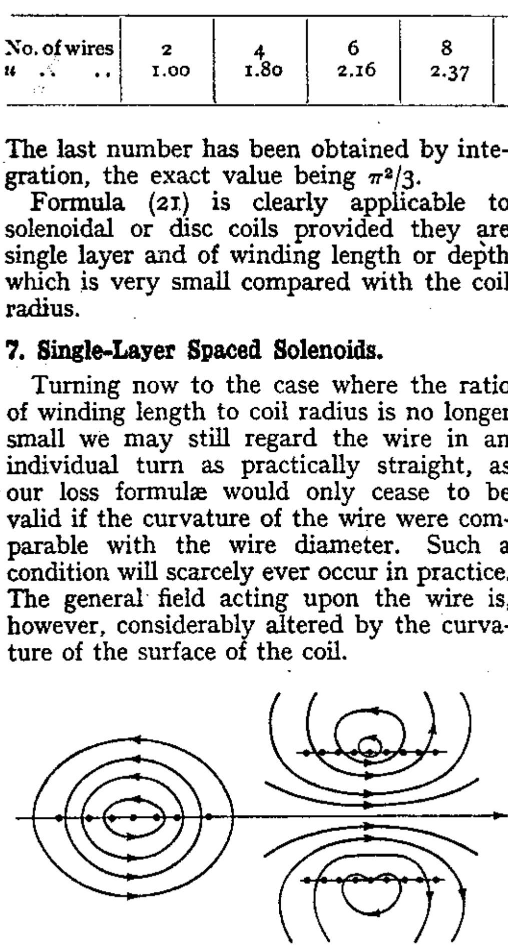
No. of wires	2	4	6	8	10	12	16	24	32	Inf.
u	1.00	1.80	2.16	2.37	2.51	2.61	2.74	2.91	3.00	3.29

The last number has been obtained by integration, the exact value being $\pi^2/3$.

Formula (21) is clearly applicable to solenoidal or disc coils provided they are single layer and of winding length or depth which is very small compared with the coil radius.

7. Single-Layer Spaced Solenoids.

Turning now to the case where the ratio of winding length to coil radius is no longer small we may still regard the wire in an individual turn as practically straight, as our loss formulæ would only cease to be valid if the curvature of the wire were comparable with the wire diameter. Such a condition will scarcely ever occur in practice. The general field acting upon the wire is, however, considerably altered by the curvature of the surface of the coil.



a b

Fig. 4 (a) *shows the symmetrical distribution of the lines of force about the central turn of a very short solenoid;* (b) *shows the distribution of the lines of force in a long solenoid when the central turn is present (upper part of figure) and when it is absent (lower part of figure).*

In the case of the straight system of wires the field acting upon one wire is perpendicular to the plane of the system, so that in very short solenoids the field is practically parallel to the coil radius, the lines of force being disposed symmetrically about the central turn as in Fig. 4 (a). As the length of the solenoid gets greater, however, the concentration of the lines in the interior of the solenoid causes the lines of force to cut across the wires of the coil in a direction inclined to the radius, so that we have to consider two components of the field, which are parallel to the coil axis and the coil radius respectively. For the central turn the axial field only is present, but for the outer turns the radial field predominates. The existence of a field parallel to the axis at the central turn will be understood when we bear in mind that the field playing on the wire is that due to the *remaining* turns. Thus (see Fig. 4), if we suppose the central turn removed, the lines of force within the coil extend into the gap as shown and thus give an axial field component through the centre of the central wire. Although this field is distorted by the field due to the current in the central wire, it is the true field to use in calculating the general field losses, as has already been explained in the preliminary survey.

In the case of an infinitely long solenoid, if the pitch of the turns is c the field inside the solenoid is $4\pi I/c$ and outside the solenoid is zero. If a turn be supposed removed, then in the neighbourhood of the gap the field changes rapidly from $4\pi I/c$ to zero as we pass through it, so that the field acting in this space may be taken as $2\pi I/c$. Using this in the loss formula (10) and proceeding as in the last section we obtain for the A.C. resistance of any one turn a formula precisely similar to (21), in which the value of u is π^2.

In the case of finite solenoids it is necessary to calculate the axial and radial field components at every point and then determine the mean square value throughout the turns. It is convenient to distinguish between the contributions of the two field components; otherwise our formula is precisely the same as (21), the following values of u holding for various ratios of coil length to coil diameter.

TABLE II.

Values of u for Single Layer Solenoids of Many Turns (spaced).

D = diameter of coil. b = length of coil.
u_1 = contribution of radial component of field.
u_2 = contribution of axial component of field.

b/D	u_1	u_2	$u = u_1 + u_2$
0.0	3.29	0.00	3.29
0.2	3.13	0.50	3.63
0.4	2.83	1.23	4.06
0.6	2.51	1.99	4.50
0.8	2.22	2.71	4.93
1.0	1.94	3.35	5.29
2	1.11	5.47	6.58
4	0.51	7.23	7.74
6	0.31	8.07	8.38
8	0.21	8.52	8.73
10	0.17	8.73	8.90
Inf.	0.00	9.87	9.87

The above values will not hold for coils of few turns, as u depends upon the turns as well as the shape of the coil. It is probably possible to calculate the value of u in these cases by assuming each turn to be a separate circle. The above values are, however, probably sufficiently accurate for purposes of design, as the approach to the final value as the turns increase appears to be fairly rapid in the case of the short coils of Section 6.

It is interesting to notice that if the length of the coil is less than its diameter u is very nearly $3.29+2b/D$.

8. Single-Layer Disc Coils.

Single layer disc coils may be treated by a method similar in principle to that for solenoids. The field is in this case entirely perpendicular to the plane of the coil. If D is the overall diameter of the coil and t is the winding depth, the factor u in formula (21) is a function of t/D which is given in Table III.

9. Closely-Wound Single-Layer Coils.

The simple formula (21) is only accurate when d/c is not too large. When the coil is closely wound and the frequency is high, the distortion of the current distribution in each turn becomes so pronounced that it is no longer permissible to regard this current as concentrated at the centre of the wire in calculating the field acting on each turn. If we recall the principle that the effect of the eddy currents is to force the magnetic field round rather than through the wire, it will be seen that in the case where the field is cutting across the wires transversely the eddy currents induced in any one turn increases the intensity of the field acting upon its neighbours, so that the simpler theory underestimates the losses. If, on the other hand, the field is tangential to the turns the eddy currents in one turn reduce the intensity of the field acting on neighbouring turns and the simple formula overestimates the losses. Thus in the case of an infinite solenoid at very high frequencies, we find, on inserting in (21) the appropriate values of u, F and G, that when the turns are practically touching, the A.C. resistance of the coil should be 5.94 times the A.C. resistance of the same wire when straight. If, however, the field due to the eddy currents is taken into account the multiplier is reduced to 3.41.

TABLE III.

Values of u for Single-Layer Disc Coils of Many Turns (spaced).

D = overall diameter of coil.
t = winding depth of coil.

t/D	u	t/D	u
0.000	3.290	0.250	4.749
0.025	3.315	0.275	5.041
0.050	3.373	0.300	5.364
0.075	3.459	0.325	5.718
0.100	3.567	0.350	6.104
0.125	3.702	0.375	6.523
0.150	3.859	0.400	6.968
0.175	4.042	0.425	7.436
0.200	4.251	0.450	7.911
0.225	4.486	0.475	8.354
0.250	4.749	0.500	8.638

The mathematical theory taking into account the external field developed by the eddy currents is rather complicated, but the writer[13] has succeeded in developing

[13] For the full theory of closely-wound coils see *Proc. Roy. Soc.*, A, Vol. 107, p. 693, 1925.
The experimental example is from Professor Howe's paper, "The High Frequency Resistance of Wires and Coils," *Journ. I.E.E.*, Vol. 58, p. 152, 1920.

relatively simple formulæ to cover the case of closely-wound coils which hold for solenoids of any length when the turns are many and for very short coils when the turns are few.

The formula for the many-turn coils is

$$R_c = R\,\{\alpha(1+F)+(\beta u_1+\gamma u_2)\,Gd^2/c^2\} \quad \ldots \ (22)$$

where α, β, γ depend both on d/c and on z as in the following Table.[14]

[14] Formula (22) is applicable to disc coils as well as solenoids, the value of u_2 being zero in this case.

as to show that it is not advisable to employ very closely-wound coils; so that the simple formula may still be used for *efficient* coils for practically all frequencies. The more elaborate formula has, however, been useful in checking the experimentally found resistances for closely-wound coils, and it has been shown that in coils where reasonable precautions have been employed in regard to the surrounding dielectric the measured resistances agree with the experimental values. In fact in a case where the losses were measured by determining the rise in

TABLE IV.

VALUES OF α, β, γ IN FORMULA (22).

d/c	$z=1$.			$z=2$.			$z=3$.		
	α.	β.	γ.	α.	β.	γ.	α.	β.	γ.
1.0	1.01	1.02	0.96	1.09	1.34	0.67	1.31	2.29	0.49
0.9	1.00	1.02	0.97	1.06	1.29	0.72	1.20	1.99	0.55
0.8		1.02	0.98	1.04	1.23	0.78	1.13	1.73	0.62
0.7		1.02	0.98	1.02	1.18	0.83	1.08	1.52	0.68
0.6		1.01	0.99	1.00	1.13	0.87	1.04	1.36	0.75
0.5		1.01	0.99		1.09	0.91	1.02	1.24	0.82
0.4		1.01	0.99		1.06	0.94	1.01	1.14	0.88
0.3		1.00	1.00		1.04	0.97	1.00	1.06	0.93
0.2					1.01	0.99		1.03	0.97
0.1					1.00	1.00		1.01	0.99

d/c	$z=4$.			$z=5$.			$z=$inf.		
	α.	β.	γ.	α.	β.	γ.	α.	β.	γ.
1.0	1.43	3.61	0.43	1.50	4.91	0.41	1.71	inf.	0.35
0.9	1.30	2.75	0.49	1.37	3.39	0.46	1.55	12.45	0.39
0.8	1.21	2.12	0.55	1.25	2.48	0.53	1.41	4.83	0.44
0.7	1.12	1.71	0.62	1.15	1.94	0.60	1.27	2.87	0.52
0.6	1.07	1.51	0.70	1.09	1.60	0.68	1.16	2.03	0.60
0.5	1.03	1.32	0.78	1.04	1.37	0.76	1.08	1.59	0.69
0.4	1.02	1.19	0.85	1.02	1.22	0.84	1.03	1.33	0.78
0.3	1.00	1.10	0.91	1.00	1.11	0.90	1.01	1.17	0.87
0.2		1.04	0.96		1.05	0.96	1.00	1.07	0.94
0.1		1.01	0.99		1.01	0.99		1.02	0.98

It will be seen that the factors α, β, γ trend towards unity as d/c and z get smaller. When z is less than unity the simple formula (21) will cover all possible values of d/c. For greater values of z, the simple formula, as will be shown below, indicates a value of d/c giving minimum losses which is such

temperature of the central turn of a fairly long solenoid, the theory shows that it is the losses due to the tangential field only that are being measured, the losses due to the transverse field, occurring as they do mainly at the ends of the coil, having very little effect upon the central temperature.

In a particular case when z was equal to 21.6, the ratio of A.C. to D.C. losses was found by the above method of measurement to be 21.8 when the spacing was such that $d/c \doteqdot 0.9$. The above formula gave the loss ratio as 27.8 when all the losses were taken into account and 22.5 when the transverse field losses were ignored. The latter value is in very fair accord with experiment, particularly when it is remembered that the theoretical formula assumes "infinite" turns. For a loosely-wound coil ($d/c \doteqdot 0.5$), theory gave the ratio of the losses as 14.1 and experiment 14.0. In this case the losses are largely the mere skin losses, and are therefore distributed practically uniformly over the coil. The example seems to indicate that the "central" temperature method of estimating the A.C. losses fails to take into account the extra end losses, and this is borne out by a consideration of the thermal constants of the coil.

10. The Correction for Self-Capacity.

In establishing the formula for the copper losses we have assumed that the current is the same in every turn of the coil. Actually this is not so at radio frequencies, because of the distributed self-capacity. As regards the reactance, the self-capacity is allowed for by assuming that the coil is shunted by a small condenser, and this assumption is justified by measurement. As regards the resistance, we can no longer appeal to experiment, but if we assume that the distributed self-capacity, is sufficiently represented by an end capacity, it is easy to show that when the reactance of the coil is large compared with the resistance the measured A.C. resistance of the coil is given by

$$R' = R_c / (1-\omega^2 LC)^2 \quad \dots \quad (23)$$

where L is the inductance of the coil, C is its self-capacity and $\omega/2\pi$ is the frequency.

As to whether this end capacity assumption is justifiable has been the subject of a great deal of discussion, but Breit[15] has shown that in the case of very short coils there is theoretical justification for the formula, and has supplied the correction which will carry us right through the resonating frequency of the coil. His corrected formula shows that (23) is good enough for most practical purposes for frequencies lower than about one-third the resonating frequency. Now, although the writer does not entirely agree in regard to the truth of Breit's formula, yet his conclusion in regard to the range of (23) is probably sound. In the neighbourhood of the resonating frequency, Breit's more elaborate formula is only valid for that portion of the copper resistance which we have represented by R_s. There is a different correction for the portion of the resistance which depends upon the mean square field acting upon the coil, as the latter does not follow the same law of distribution as the mean square current when we take into account the variation of current from turn to turn of the coil due to the flowing off of the dielectric current. It is not proposed, however, to deal with this in the present article.

It will be seen from formula (23) that it is a distinct advantage to use coils of low self-capacity even from the point of view of copper losses, as the circulating currents due to the self-capacity cause loss of energy in the copper portions of their paths. This added resistance due to self-capacity is thus a true copper loss. It is sometimes found that stray wires attached to coil terminals cause an appreciable increase in resistance of the coil. This may be in part accounted for by the resulting increase in the apparent self-capacity of the coil, especially when the coil is being used near its own natural frequency.

11. Single-Layer Coils at High Frequencies.

When the frequency is so high that the approximations (9) and (14) hold for $1+F$ and G, formula (21) assumes the simple form

$$R_c = \frac{A}{\sqrt{\lambda}} + B \quad \dots \quad \dots \quad (24)$$

where λ is the wave-length in metres and, for copper, A and B have the values

$$A = 660\, Rd \left(1 + \tfrac{1}{4}u\frac{d^2}{c^2}\right)$$
$$B = \tfrac{1}{4}R \left(1 - \tfrac{1}{2}u\frac{d^2}{c^2}\right) \quad \dots \quad (25)$$

Now, in such cases it has been found experimentally that the measured effective resistance, after due correction for self-capacity, fits very closely a formula of the type

[15] *Bureau of Standards*, Scientific Paper No. 430, 1922.

$$R_m = \frac{A'}{\sqrt{\lambda}} + \frac{B'}{\lambda^2} \quad \cdots \quad \cdots \quad (26)$$

and the value of A' required to fit (26) agrees closely with the value of A in (24). We need not trouble much about the term B in (24) as it is usually so small that it is within the error of measurement. There seems to be no doubt therefore, that the first term in (26) is that due to the copper loss in the coil. As regards the second term it must be noted that in all cases where a formula of this type has been found to fit the experimental facts, the method of measurement employed has been the well-known resistance variation method, in which the coil forms part of a resonating circuit containing an air condenser which is assumed free from loss. It may be that a portion of this term is really due to loss which does not occur in the coil at all. This view seems to be supported by the few reliable measurements which have been made in which the method does not involve a resonating condenser. There is not, however, sufficient evidence at present to decide this question.

As regards the relative values of the two terms in (26) the following experimental results are of interest: Table V. gives the details of measurement and analysis in the case of a single-layer coil of 23.5 cm. diameter and winding length 1.0 cm. wound with 13 turns of No. 22 D.S.C. wire, the whole being held together by wax and silk tape. The measured inductance of the coil was 100μH and the self-capacity 20$\mu\mu$F.

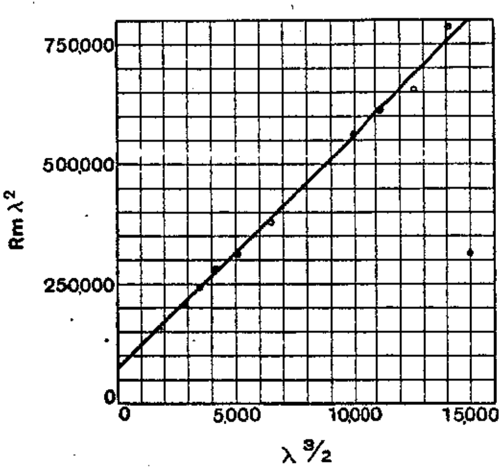


Fig. 5. *Showing how* A′ *and* B′ *in the formula* $R_m = \frac{A'}{\sqrt{\lambda}} + \frac{B'}{\lambda^2}$ (26) *may be found by plotting* $R_m \lambda^2$ *against* $\lambda^{3/2}$ *and finding the slope* (A′) *and the intercept on the vertical axis* (B′) *of the resulting straight line.*

The values of A' and B' used in the Table are

$$A' = 49.7, \; B' = 7.0 \times 10^4.$$

TABLE V.

Separation of Losses in Single-Layer Coil.

R_1 = measured A.C. resistance. $R_m = R_1(1-\omega^2 LC)^2$. (See formula 23.)
$A'/\sqrt{\lambda}$ = term due to copper loss. B'/λ^2 = term due to remaining losses.

The values of A' and B' are found by plotting $R_m\lambda^2$ against $\lambda^{3/2}$ as in Fig. 5. If (26) is valid the result should be a straight line the slope of which determines A' and the intercept on the vertical axis determines B'. The figure and the smoothed values of R_m show that the law holds for this coil.

λ (Metres.)	R_1 (Ohms.)	R_m (Ohms.)	$A'/\sqrt{\lambda}$	B'/λ^2	Smoothed R_m (Ohms.)	Cu. loss. / Total loss.
206	7.2	5.00	3.40	1.65	5.05	0.67
229	6.2	4.64	3.30	1.34	4.64	0.71
260	5.2	4.16	3.09	1.02	4.11	0.75
294	4.3	3.61	2.90	0.81	3.71	0.78
350	3.5	3.10	2.66	0.57	3.23	0.82
463	2.8	2.62	2.31	0.33	2.64	0.87
500	2.6	2.46	2.22	0.28	2.50	0.89
543	2.5	2.38	2.14	0.24	2.38	0.90
584	2.3	2.30	2.06	0.20	2.26	0.91

The D.C. resistance of this coil was 0.444 ohm, so that we find from (25)

$$R_c = \frac{51}{\sqrt{\lambda}} - 0.05$$

Thus the term depending upon wave-length agrees very closely with the value deduced from the measurements, while the constant term is very small.

The values for six other coils are given in Table VI. below.

12. Effect of Spacing in Single-Layer Coils.

Suppose a single layer coil to be wound with various gauges of wire, keeping the pitch of the turns constant. Under these circumstances the inductance would remain practically constant, but an examination of (25) shows that the resistance will pass through a minimum value for a particular gauge of wire. To prove this, notice that since R is inversely proportional to the square of the wire diameter, the quantity A is proportional to $\frac{1}{d} + \frac{1}{2} u \frac{d}{c^2}$. Thus A is equal to the sum of two terms which have a product independent of the diameter.

TABLE VI.

Coil.	D cm.	b cm.	d mm.	c mm.	Inductance (μH.)	Observed A'	Observed B'	Calcd. A'	B'/L^2.
1	31	1.8	2.2	2.6	36	9.5	2.14×10^4	10.4	16.4
2	19.5	2.6	2.0	2.4	43	11.8	2.15 ,,	11.2	11.5
3	22.6	2.0	0.7	1.5	75	25.1	7.4 ,,	25.5	13.1
4	20.0	6.5	3.0	3.6	80	14.3	7.0 ,,	13.2	10.9
5	19.6	2.15	1.1	1.4	84	25.5	7.0 ,,	25.6	10.0
6	23.5	1.0	0.7	0.8	100	50	7.0 ,,	51.0	7.0
7	17.5	7.3	1.62	1.8	320	56	1.7×10^6	55.0	16.5

It is interesting to notice that if B is divided by the square of the self-inductance of the coil, as in the last column, the result is of the same order of magnitude for all the coils. This is the result we should expect if the coils were all shunted by a constant high resistance of the order of 0.2 to 0.4 megohm.

The values for coils 3, 6 and 7 are from measurements taken at the N.P.L. about six years ago using the best condensers then available. It is understood that since then better condensers have been constructed, and it would be of interest to have a corresponding series of values with these new condensers.

The remaining coils were measured by Lindemann and Hüter.[16]

Although great stress has been laid in this section upon the loss outside the copper, the copper loss is usually the predominant factor, as is seen from the last column of Table V. The coil of this Table is one which would normally be used at a wave-length in the neighbourhood of 350 metres, so that then the copper losses are 82 per cent. of the total losses.

[16] *Verh. Deutsch. Phys. Gesellschaft*, Vol. 15, p. 219, 1913.

The two terms may therefore be regarded as the two sides of a rectangle, and variation of the diameter is equivalent to alteration of the shape of this rectangle keeping its area (product of sides) constant. The sum of the two sides is half the periphery of the rectangle, so that the periphery of the rectangle measures the size of the quantity A. Now, for rectangles of equal area, the one with the least periphery is a square. The value of A is then the least when

$$\frac{1}{d} = \frac{1}{2} u \frac{d}{c^2}$$

or

$$\frac{d}{c} = \sqrt{\frac{2}{u}} \quad \ldots \quad \ldots \quad (27)$$

Now, at very high frequencies, (24) shows that the copper loss is proportional to A, so that the condition of minimum loss is then given by (27). Using this value of d in (24) and neglecting B we find that the minimum value of the copper resistance at very high frequencies is given by the formula

$$R_c = 0.0020\frac{l}{c}\sqrt{\frac{u}{\lambda}} \qquad .. \quad (28)$$

the D.C. resistance having been expressed in terms of the length l of the wire, the resistivity of copper (1.7×10^{-6} ohms cms.) and the diameter of wire as expressed by (27). R_c is in ohms when l, c are in cms, and λ is in metres.

In support of this theory we may quote the following experimental results*: A set of coils were wound on formers each of diameter 3 in. and winding length 2 in. and each having 53 turns. The gauges of wire varied from coil to coil, and curves were obtained connecting d/c with resistance for various wave-lengths. Thus for the case of $\lambda = 300$ metres the following smoothed values were read from the curves.

d/c ..	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
R_m	12.5	7.4	5.4	4.7	4.6	4.8	5.5	7.1	ohms.

* The writer is indebted to the Federal Telegraph and Telephone Co., New York, for permission to make use of these results.

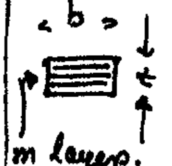
Now, for this coil $u=4.62$, so that by (27) the best value of d/c should be 0.66 and then by (28) the copper resistance is 3.3 ohms. The best spacing is in close agreement with experiment, and the value of the copper resistance is then 70 per cent. of the measured value.

The curves for 200 and 500 metres show similar minima at $d/c=0.6$ and the minimum values indicate that the copper losses as given by (28) are 65 and 78 per cent. respectively of the total losses. These percentages are of the same order as those discussed in the last section.

As regards the measured resistances at closer spacings, the theoretical formula (21) does not give as rapid an upward trend as the curves indicate, but comparison with the more elaborate formulæ for close windings shows that this upward trend may still be mainly due to copper loss.

Theory and experiment therefore show that formula (27) is reliable for determining the best gauge of wire to use for a single-layer coil of any given shape. The question of the best shape of coil is deferred until we have dealt with the formulæ for multilayer coils.

Effective Resistance of Inductance Coils at Radio Frequency.—Part III.



By S. Butterworth.

(Admiralty Research Laboratory, Teddington.)

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13. Multilayer Coils of Small Winding Depth.

Let the winding section of the coil be $b \times t$. Let t/b be small and suppose at first that b is small compared with the radius of the coil. Let there be m layers in the depth t and n turns per layer.

The field at any point in the section will have two components, H_t and H_b, parallel to t and b respectively, which will act independently in producing eddy losses.

As regards H_t, the field acting on a single wire is the same as that for a single layer coil for which $c=b/mn$. Thus the added resistance due to the action of H_t is

$$\tfrac{1}{3}\pi^2 RG(mnd/b)^2$$

the number of turns being assumed large.

As regards H_b, each layer behaves as a current sheet having current density nI/b. In the immediate neighbourhood of the sheet the component of the field parallel to the sheet is

$$h = 2\pi nI/b$$

and reverses its direction as we pass through the sheet.

If, as is assumed, t/b is small, the value of h due to any one layer will remain the same throughout the winding section, so that the field acting on the top layer due to the remaining layers is $(m-1)\,h$, on the next layer $(m-3)\,h$, on the third layer $(m-5)\,h$ and so on.

The mean square field for all the layers is therefore

$$h^2\{(m-1)^2+(m-3)^2+(m-5)^2+\ldots\}/m$$
$$= (m^2-1)h^2/3 = \tfrac{4}{3}\pi^2(m^2-1)(nI/b)^2.$$

Applying this result to the eddy loss formula it follows that the added resistance due to the action of H_b is

$$\tfrac{1}{3}\pi^2(m^2-1)RG(nd/b)^2$$

The total resistance of a short shallow multilayer solenoid or disc coil is therefore got by adding these two resistance terms to the skin resistance and we have

$$R_c=R\{1+F+\tfrac{1}{3}\pi^2(2m^2-1)G(nd/b)^2\} \quad (29)$$

The same reasoning may be applied to the case where b is not small compared with the radius of the coil and it is found that

$$R_c=R\left\{1+F+\left(u+\tfrac{1}{3}\pi\frac{2m^2-1}{m^2}\right)G(mnd/b)^2\right\} \quad (30)$$

In applying to solenoidal coils, the length b is interpreted as the winding length of the coil and in applying to disc coils b is the difference between the inner and outer radii. In either case, the width t is supposed to be very small.

If the number of layers is large, and the total number of turns $(m \times n)$ is N, formula (30) becomes

$$R_c=R\{1+F+(u+\tfrac{1}{3}\pi^2)G(Nd/b)^2\} \quad (31)$$

14. Multilayer Coils of Finite Winding Depth.

The theory of the losses in multilayer coils at very high frequencies has been given by Prof. Fortescue[17] who, however, confined himself to coils for which the winding length lies between one and three times the overall radius, and, for solid wire coils, assumed the wire so thick that the square root law holds in regard to frequency. Otherwise his method is similar to that developed above. As regards the mean square field acting on the wires of the coil, Fortescue expresses this in the form

$$H^2_m = K^2N^2I^2/D^2 \quad .. \quad (32)$$

where

D = overall diameter of coil.
N = total turns,

and K is a factor depending on the ratios

[17] *Loc. cit.* [11].

b/D and t/D in which b is the winding length and t the winding depth. Tables of values of K are given covering the range $b/D=0.5$ to $b/D=1.5$.

It turns out, however, that these longish coils are uneconomical both in regard to space, number of turns and length of wire, so that the writer considered it advisable to calculate the values for shorter coils and hoped that at the point of overlapping the values would link on to those of Prof. Fortescue. This, unfortunately, was not so, the discrepancy being such that in extreme cases the copper losses using Fortescue's values of K are only 60 per cent. of those obtained with the writer's values. Some means of checking which of the values was the more reliable was therefore necessary.

The mode of arriving at the K factors is long and tedious, and the writer was not prepared to spend the time recalculating the values over the range of Fortescue's coils. The check adopted was therefore to find whether the present or Fortescue's values trended satisfactorily towards the easily found values for infinitely long coils.

The factor K becomes zero for such coils, but if we multiply K by b/D the limiting value for b/D infinity is finite. The method of check adopted therefore was to plot curves for the various values of t/D of the factor Kb/D, using D/b as abscissæ, and to find which set of values was pointing to the proper value of Kb/D when $D/b = 0$.

These curves, using the *present* values, are given in Fig. 6. It is seen that the portions of the curves linking the last calculated values of Kb/D for finite coils ($D/b=2$) to the zero value are continuous with the remainder. This could not be done with Fortescue's values.

We are forced to conclude that the present values must be the correct ones. The curves of Fig. 6 may be used to get approximate values of K for any length of coil.

The formula for Kb/D for infinitely long coils is

$$Kb/D = 2\pi\{(4D-6t)(3D-3t)\}^{\frac{1}{2}} \quad \ldots (33)$$

For short coils the values of K as found by the present writer are given in Table VII. In comparing with Fortescue's Table it should be noted that Fortescue uses the ampere as the unit of current, so that apart from the above mentioned discrepancy Fortescue's K values are ten times the present values.

For the case of solenoids, for which $t/D=0$, it is easily shown by the theory of the previous section that

$$K^2 = \frac{4D^2}{b^2}\left\{u + \tfrac{1}{3}\pi^2\left(1 - \frac{1}{m^2}\right)\right\} \quad \ldots (34)$$

and a similar formula holds for disc coils for which $b/D=0$. The mode of arriving at the values of K in the general case is indicated in the Appendix.

We can now readily arrive at the A.C. resistance formula for multilayer coils by using the value of H^2_m given by (32) in the eddy loss formula (10) and adding this loss to the ordinary skin loss. We then obtain

$$R_c = R\left\{1 + F + \tfrac{1}{4}\left(\frac{KNd}{D}\right)^2 G\right\} \quad \ldots (35)$$

TABLE VII.

VALUES OF THE FACTOR K IN FORMULA (32).

b/D	0.000	0.125	0.250	0.375	0.500
t/D					
0.0	Inf.	41.7	21.2	14.4	11.0
0.1	52.4	23.3	15.4	11.6	9.5
0.2	27.4	16.2	12.4	9.9	8.2
0.3	19.6	13.7	10.7	8.8	7.5
0.4	16.0	12.0	9.5	8.0	6.9
0.5	13.8	10.4	8.4	7.0	6.0

NOTE.—The column $b/D=0$ refers to *many* layered disc coils, and the row $t/D=0$ to *many* layered solenoids. When the layers are few the following values hold for K:—

SOLENOID WITH m LAYERS.

b/D	0.000	0.125	0.250	0.375	0.500
m					
1	—	30.1	15.6	10.7	8.3
2	—	39.2	20.0	13.6	10.4
3	—	40.6	20.7	14.0	10.7
Inf.	—	41.7	21.2	14.4	11.0

DISC COIL WITH m LAYERS.

t/D	0.1	0.2	0.3	0.4	0.5
m					
1	37.8	20.6	15.4	13.2	11.7
2	45.0	25.9	18.6	15.3	13.3
3	51.0	26.8	19.2	15.7	13.6
Inf.	52.4	27.4	19.6	16.0	13.8

This formula, together with Tables I. and VII. and the curves of Fig. 6, includes all the previous formulæ and tables and constitutes the final solution of the A.C. resistance problem for solid wire coils with well spaced windings.

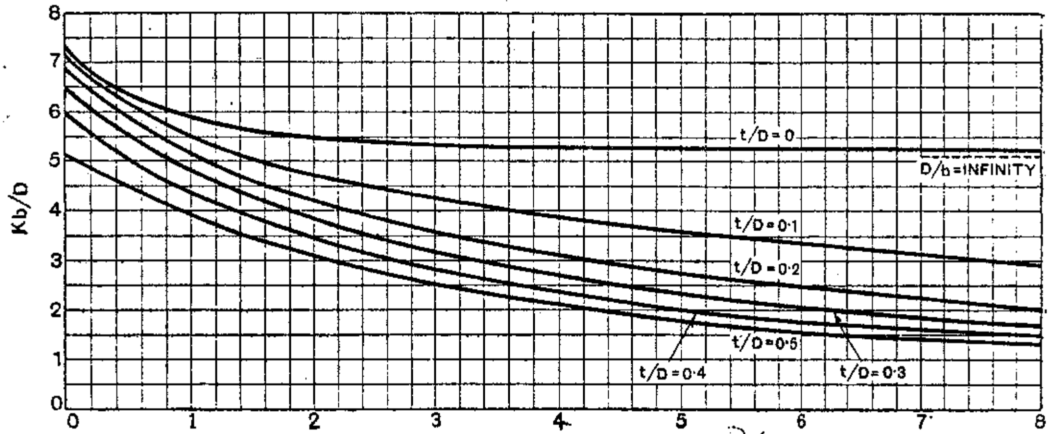

Fig. 6.

In application the self-capacity correction as given by (23) must also be made when the coil is worked at wave-lengths which are comparable with the natural wave-length of the coil.

15. Test of Formula by Comparison with Observed Resistances.

Our first example is a coil in which there is no space factor other than that due to the unavoidable insulation space. In this case the general field losses are the main term in the resistance, so that the comparison gives a very severe test of the general field loss formula.

A coil of 5,000 microhenries was wound with 350 turns of No. 24 D.S.C. wire, the dimensions of the coil being $D=7.5$ cm., $b=0.75$ cm., $t=2.5$ cm.

The D.C. resistance of the coil was 3.8 ohms and the following A.C. resistances were measured by the reactance-variation method in a resonant circuit :—

Wave length (metres).	Measured resistance (ohms).	Calculated copper resistance (ohms).	Cu. loss / Total loss.
2,000	370	322	0.87
2,500	283	242	0.86
3,000	222	193	0.87

The second example is for a series of coils of equal dimensions but of different inductances, each coil being measured at that wave-length which gives resonance with a condenser of 1,000$\mu\mu$F. The common dimensions are $D=8.3$ cm., $b=1.5$ cm., $t=3.0$ cm.

Coil No.	Turns.	Wire gauge.	Inductance (μH.)	Wave-length (metres).	Resistances. D.C.	Resistances. A.C.		Cu. loss / Total loss.
					Ohms.	Obsd.	Calc.	
1	40	22	79	540	0.33	2.5	2.1	0.84
2	72	22	197	840	0.53	5.1	4.4	0.86
3	98	23	364	1,150	0.88	9.0	7.4	0.82
4	141	24	745	1,650	1.58	17.4	15.2	0.87
5	244	24	2,240	2,860	2.75	43.5	40.0	0.92
6	374	28	5,280	4,350	9.61	29.2	24.8	0.85
7	570	30	12,400	6,700	20.8	48.9	33	0.68
8	835	30	26,300	9,800	31.3	69.5	48	0.69
9	1,280	34	62,200	15,000	85	127	95	0.75
10	2,150	36	177,300	25,000	190	204	204	0.77

The wire gauge and turns have been estimated from the measured D.C. resistance and inductance and the calculated A.C. resistance is based on these estimated values. There were also small variations in the measured dimensions, but for ease of calculation the average values of D, b and t throughout the series was used. Thus the loss ratio in the last column can only be taken as representing approximately the importance of the copper loss.

Other examples will be given later in illustrating points of design. The above examples are however sufficient to show that the basis of design must be that for minimum copper loss and not (as many people suppose) from the point of view of dielectric loss.

16. Design of Inductance Coils for Minimum Copper Losses—General Principles.

The general problem to be solved is to find what form of coil to use and what diameter of wire to employ to produce a coil having a given inductance and the smallest possible resistance at a given frequency. In regard to the form of the coil, factors other than the resistance and inductance also enter into the problem. Thus if we are considering a frame aerial the turn area enters into the question, as this determines the E.M.F. induced by a given external field. It is also sometimes considered that a flat type of coil is advantageous, as then we may obtain close coupling with another similar type of coil. It has even been suggested that the type of coil should be based upon the ease with which it may be adapted to existing types of plugs and plug holders. Relative costs of construction also play an important part in deciding between the relative merits of two coils so that, other things being equal, a coil which will give the requisite inductance with few turns is to be preferred to one requiring many turns, as the former coil will be quicker to wind. It is clearly therefore impossible to lay down a definite rule and say that a coil of any particular shape is the best coil for all purposes.

We may, however, prescribe certain conditions to fix our problem and find the "best" coil fulfilling these conditions.

For the purpose of comparing coils of different shapes we will take three different assumptions as follows :—

(*a*) The volume of the copper is the same for all coils.

(*b*) The same length of wire is used in all coils.

(*c*) Each coil occupies the same space.

The first method of comparison is equivalent to the old D.C. problem in which a given amount of copper was available and it was required to utilise this copper to give the best D.C. time constant (ratio of inductance to resistance). The resulting shape of coil is sometimes referred to as the most economical shape. This would only be true if the cost of the copper were the main cost of the coil or if the other costs remained constant. For the second method of comparison to be fair, the coil costs must be mainly proportional to the length of the wire. This seems fairly reasonable, especially in the case of stranded wire coils where the wire is expensive, and also takes into account the fact that the cost of winding depends largely on the length of wire. The third method is of value when compactness is an important consideration. A small difficulty arises in deciding how to define "space occupied." If we express this by mere overall volume of coil our equations lead us to a single layer disc coil of very large diameter, but of negligible length, a form which is by no means compact. If we measure space occupied by the overall diameter (as was done by Prof. Fortescue) we are led to a very long solenoid. Prof. Fortescue got out of this difficulty by pointing out that after a certain length the gain in resistance was small and therefore recommended a coil of length equal to the diameter. If, however, we wish to give both length and diameter due weight the most suitable criterion appears to the writer to compare coils of equal surface area, and the Tables to be given refer to coils satisfying this condition. The coils compared on this basis therefore fall in diameter as the length increases, and the disc coil is kept of finite diameter. If sketches of a series of such coils be drawn they appear to the eye to be about equally compact, except perhaps for the long, small-diameter solenoids. However, it turns out that we have passed through the region of efficient coils before

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17. Inducta

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b/D	0.0
t/D	
0.0	In
0.1	17.
0.2	11.
0.3	7.
0.4	5.
0.5	3.

18. Design f

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reaching this stage. This mode of comparison may seem somewhat arbitrary, but it is necessary to make some such assumption to put the matter upon a numerical basis. In addition, it turns out that if the comparison be made on assumption (*a*), (*b*) or (*c*) the resulting best shape is nearly the same, and the difference between coils in the neighbourhood of the " best " shape are so slight in regard to increase of resistance that the designer is left with considerable latitude to fulfil conditions of good coupling or of accommodation into various shapes of spaces without thereby departing seriously from the best resistance conditions.

As regards the equally compact coils (condition (*c*)) the vertical columns of the Tables are absolutely valid and tell us the best depth of winding to employ for any given ratio of length to overall diameter.

17. Inductance of Coil.

As we are going to compare coils of equal inductance, we require an equation connecting the inductance with the dimensions and turns of the coil. The equation for the inductance will be written

$$L = \frac{L_0 N^2 D}{1,000} \quad .. \quad .. (36)$$

in which L_0 is a shape factor and is given by the following Table, and L is the required inductance in μH.

TABLE VIII.

VALUES OF L_0.

b/D	0.000	0.125	0.250	0.375	0.500
t/D					
0.0	Inf.	18.68	14.43	12.02	10.37
0.1	17.46	12.92	10.52	8.93	7.78
0.2	11.51	9.10	7.58	6.49	5.68
0.3	7.82	6.33	5.31	4.57	4.00
0.4	5.26	4.27	3.59	3.08	2.69
0.5	3.48	2.82	2.37	2.03	1.78

18. Design for Given Volume of Copper.

We will first compare coils which are all wound with the same length and diameter of wire. If the length of the wire be l we have

$$l = \pi N(D - t) = \pi ND\,(1 - t/D) \; ... \quad (37)$$

Now for each possible shape of coil equation (36) will determine N^2D in terms of the inductance, and equation (37) will determine ND in terms of the length of wire, so that the two equations may be used to find N and D separately. As we pass from one shape of coil to another the values of N and D will vary, as (36) and (37) involve shape factors. Now in the resistance equation (35) the D.C. resistance will be the same for all the coils and also the functions F and G will be the same. In fact the only quantity that varies for different shapes of coils is the factor KN/D. The best shape of coil is clearly therefore the one which yields the minimum value for KN/D. If we use (36) and (37) to express N and D in terms of shape factors we find that KN/D is proportional to $K(1 - t/D)^3/L_0^2 = \phi$ say.

The values of ϕ are given in Table IX.

The Table shows that the best coil is a single-layer coil having a winding length equal to one-third its diameter. The best single-layer disc coil should have a winding depth equal to one-quarter the external diameter. If a multilayer coil is desirable the Table shows that there is a wide range of choice, and if we make $5t + 3b = D$ we shall never depart greatly from the condition of maximum efficiency.

TABLE IX.

VALUES OF $\phi = K\,(1 - t/D)^3/L_0^2$.

S.L. = Single-layer. M.L. = Multi-layer.

b/D	0.000		0.125	0.250	0.375	0.500
t/D						
0		S.L.	0.087	0.075	0.075	0.077
		M.L.	0.120	0.102	0.100	0.102
	S.L.	M.L.				
0.1	0.090	0.125	0.102	0.101	0.106	0.114
0.2	0.080	0.106	0.101	0.110	0.120	0.130
0.3	0.079	0.110	0.118	0.130	0.144	0.161
0.4	0.101	0.124	0.142	0.158	0.182	0.206
0.5	0.120	0.142	0.163	0.187	0.212	0.230

Next, keeping the shape constant, let us suppose a series of coils wound with different diameters of wire, the length of wire being chosen so that the volume of copper remains constant; that is, l must be proportional to $1/d^2$. Again the number of turns and the overall diameter will vary from coil to coil. The inductance equation (36) shows that N^2D remains constant and (37) shows that ND is proportional to l. Hence N is proportional to $1/l$ and D to l^2 so that N/D is proportional to $1/l^3$, that is to d^6. In the resistance equation (35) the D.C. resistance R varies as l/d^2 or as $1/d^4$. The functions F and G also vary. For *low* frequencies F is negligible and G varies as d^4.

Then from these variations and (35), the skin resistance R_s $\{= R(1 + F)\}$ is of the form A/d^4 and the general field resistance R_h is of the form Bd^{14} where A and B are independent of diameter.

The whole resistance may therefore be written

$$R_c = R_s + R_h = \frac{A}{d^4} + Bd^{14} \qquad (38)$$

The best diameter of wire is found by differentiating (38) with respect to d and equating to zero. Doing this we find

$$2R_s = 7R_h \quad \dots \quad \dots \quad (39)$$

For *high* frequencies F and G are large and both proportional to d. In this case R_s is of the form A'/d^3 and R_h of the form $B'd^{11}$. The minimum occurs when

$$3R_s = 11R_h \quad \dots \quad (40)$$

The difference between conditions (39) and (40) is so slight that we may say that for *any* frequency the best diameter is such that $R_s = 3.6R_h$.

The practical method of deducing the proper diameter is deferred to a later section.

19. Design for Fixed Length of Wire.

If coils of equal length of wire are compared, the best shape is found exactly as in the last section. The best diameter is however different, as we now suppose the different coils wound with the same length of wire in all cases.

R now varies as $1/d^2$ and for a given shape N/D remains constant. Hence at *low* frequencies R_s is proportional to $1/d^2$ and R_h to d^4 so that by the method of the previous section the condition for the best diameter becomes

$$R_s = 2R_h \quad \dots \quad \dots \quad (41)$$

At *high* frequencies R_s is proportional to $1/d$ and R_h to d and the best diameter condition is

$$R_s = R_h \quad \dots \quad \dots \quad (42)$$

20. Design for Fixed Overall Surface.

The overall surface S of the coil is given by the formula

$$S = \tfrac{1}{2}\pi D^2 (1 + 2b/D) \quad \dots \quad (43)$$

It is clear that if we take a coil of given shape and inductance the condition of constant surface fixes the diameter of the coil and the inductance then fixes the number of turns. Hence the length of wire is fixed and the best diameter of wire is therefore the same as in the last section.

The best shape of coil depends to some extent upon the frequency.

For *low* frequencies the condition $R_s = 2R_h$ makes d^3 proportional to D/KN, and when the condition is satisfied

$$R_c = 6\rho N (D - t)/d^2 \quad \dots \quad (44)$$

or on elimination of d, Rc is proportional to

$$(1 - t/D)\ (K^2N^5D)^{\frac{1}{3}}$$

The inductance equation (36) and the surface equation (43) enable us also to express N and D in terms of shape factors, and we obtain finally

$$R_c = A\,K^{\frac{1}{3}}\,(1 - t/D)\,(1 + 2b/D)^{\frac{1}{3}}/L_0^{5/6} \quad (45)$$

in which A is proportional to $f^{\frac{1}{3}}\,L^{5/6}/S^{\frac{1}{3}}$.

By a similar process of reasoning in the case of *high* frequencies we obtain

$$R_c = BK\,(1 - t/D)(1 + 2b/D)^{\frac{1}{2}}/L_0 \quad (46)$$

in which B is proportional to $f^{\frac{1}{2}}\,L/S^{\frac{1}{2}}$

To compare various shapes of coils we must therefore calculate the shape factors (45) and (46). The results are given in Table X.

On comparing these Tables with Table IX., it is seen that the best coils for a given space are somewhat longer than the best coils for a given amount of copper, but, owing to the slow variation in the neighbourhood of minimum resistance, the same general rules may be taken to apply in both cases. The advantage of single layer coils is again apparent.

A study of the numbers in the vertical columns is instructive, as the numbers in any one column refer to coils of equal outside surfaces and we therefore see how far it is profitable or otherwise to increase the winding depth. Thus if we have decided that it is convenient to use a winding length equal to one-quarter the overall diameter, the column $b/D = 0.25$ tells us the best resistance we can get for various winding depths.

The first four numbers show that as we pass from single to multilayer coils, keeping the winding depth very small, there is a progressive increase in resistance. The remaining five numbers give the effect of diminishing the internal diameter keeping the external diameter constant. There is a diminution in resistance until $t/D = 0.15$ and then the resistance increases again, but the gain obtained by increasing the winding depth is not enough in this case to counterbalance the loss in passing from a single layer to a multilayer coil.

It must be emphasised that these comparisons refer to coils of equal inductance, so that for each winding depth the turns are always adjusted to satisfy this condition; also the diameter of wire in each case is such as to give the minimum resistance for each shape. In this respect the comparison differs from that usually made in comparing various shapes of coils, where it is nearly always assumed that the diameter of the wire is constant.

21. Large and Small Coils compared.

The forms of the A and B factors in (45) and (46) enable us to answer the question as to what advantage may be expected by increasing the diameter of the coil.

Since S is proportional to D^2 for a given shape, the form of A shows that for *low* frequencies the "best" resistance varies inversely as the square root of the diameter and for *high* frequencies the "best" resistance varies inversely as the diameter as is seen from the form of B.

The reason for the slow increase in efficiency with increase of coil diameter at low frequencies is seen when we consider the variation of "best" space factor with coil diameter.

The area of winding section available per turn is proportional to D^2/N, and, if the inductance is held constant, N^2 is

TABLE X.

RELATIVE RESISTANCES OF COILS OF EQUAL INDUCTANCE AND EQUAL SURFACE.

(A) LOW FREQUENCIES.

b/D	0.000		0.125	0.250	0.375	0.500	0.625	0.750	0.875
t/D									
0.0	one layer		0.89	0.75	0.71	0.70	0.70	0.71	
	two layers		1.07	0.88	0.83	0.81	0.80	0.81	
	three layers		1.09	0.90	0.84	0.82	0.82	0.83	
	multilayer		1.11	0.92	0.86	0.84	0.83	0.84	
	S.L.	M.L.							
0.1	0.96	1.19	0.92	0.87	0.86	0.88	—	—	
0.2	0.79	0.95	0.86	0.88	0.89	0.90	—	—	
0.3	0.78	0.92	0.91	0.94	0.97	1.00	—	—	
0.4	0.84	0.95	0.99	1.03	1.08	1.13	—	—	
0.5	0.91	1.02	1.06	1.11	1.17	1.21	—	—	

(B) HIGH FREQUENCIES.

b/D	0.000		0.125	0.250	0.375	0.500	0.625	0.750	0.875
t/D									
0.0	one layer		1.81	1.32	1.18	1.12	1.12	1.12	1.14
	two layers		2.36	1.68	1.49	1.40	1.39	1.38	1.39
	three layers		2.44	1.74	1.44	1.43	1.42	1.42	1.43
	multilayer		2.51	1.79	1.49	1.49	1.45	1.45	1.49
	S.L.	M.L.							
0.1	1.95	2.70	1.82	1.60	1.54	1.55	—	—	—
0.2	1.43	1.90	1.59	1.59	1.61	1.64	—	—	—
0.3	1.20	1.76	1.70	1.72	1.78	1.80	—	—	—
0.4	1.50	1.82	1.88	1.93	2.06	2.17	—	—	—
0.5	1.68	1.98	2.06	2.10	2.27	2.37	—	—	—

proportional to $1/D$. Hence as the coil diameter increases the area commanded by a turn varies as $D^{5/2}$. On the other hand the best diameter of wire is proportional to $(D/N)^{\frac{1}{2}}$ that is to $D^{\frac{1}{2}}$ when the inductance is held constant, so that the copper area only increases as the diameter of the coil.

Thus although there is a rapid increase in available space for the turns we cannot take advantage of this as we would increase the A.C. resistance of the coil by filling the space with copper. The only way to make further use of the available space is to use stranded wire instead of solid wire.

NOTE.—Since the publication of the previous instalment of this article, Mr. A. L. M. Sowerby has published an article in which inductance coils of various forms are compared experimentally (*E.W. & W.E.*, June, 1926). He arrives at the conclusion that there is a "best" spacing but not a "best" ratio of length to diameter. His results are not, however, in contradiction with theory as the coils prepared by him all had equal wire diameters. In Table I. of Mr. Sowerby's paper it is shown that the H.F. resistance remained constant for a large variation of length-diameter ratio. Fortunately, Mr. Sowerby also adds a column giving the D.C. resistance of the various coils, which is shown to pass through a definite minimum. Since the diameter of the wire is constant, this means that the length of wire employed passes through a minimum, and therefore there is a "best" shape, viz., that which will produce the coil with the minimum amount of wire. Similarly, if we calculated the spaces occupied by each coil as defined by condition (*c*) above, we should find a "most compact" coil in Mr. Sowerby's series. These remarks seem to emphasise how careful one must be in deciding what is the best shape of coil, and shows why so many different recommendations have been made from time to time.

S. BUTTERWORTH.

12th June, 1926.

Among the Experimental Transmitters.

THE indiscriminate output of QSL cards by listeners is already becoming a nuisance to those amateur transmitters who are engaged in serious research. A great number of these cards undoubtedly fulfil the desired object of furnishing transmitters with useful reports concerning their tests, but we fear that an ever-increasing number is sent out merely with a view of collecting "wallpaper," and resembles the quest by small boys for cigarette cards. The senders of these worthless cards appear aggrieved if the recipients fail to reply. One amateur informs us that if he were to send a reply to every QSL card he receives, it would be necessary for him to engage a secretary for that purpose only, and the cost in postage would be very considerable.

Perhaps the best solution to this difficulty would be for transmitters to devise a code which would signify the nature of the reports desired. An experimenter working on comparatively high power would probably only be interested in reports from distant stations, while one conducting tests on low-power might welcome cards from listeners at all ranges; another might only be interested in fading effects or meteorological observations, and cards merely stating that his signals had been heard would be of no material use. We suggest that the T. & R. Section of the R.S.G.B. should consider this matter and arrange some such code. Listeners who wish to help experimenters will then know what special records are desired, while mere collectors of wallpaper will tacitly be warned that no contributions to their collections may be expected from that particular source.

Norwegian and Danish Amateurs.

We understand from correspondents in Norway and Denmark that amateurs in those countries are now able to obtain transmitting licences.

In Norway licences are being issued for transmission on 3—6, 29—35, 43—47, 69—75 and 100—120 metres. The aerial power must not exceed 20 watts. The licencee must possess certain technical qualities and be able to send and receive at least 12 words per minute. The annual fee is fixed at 30 kroner. QSL cards should be sent via the Norsk Amatoer Sender Union, Oslo. The nationality prefix adopted by Norwegian amateurs is LA.

Our Danish correspondent states that the maximum input allowed to amateurs is 100 watts, and the wavelengths allotted are: Below 15 metres, 43—47, 70—75 and 95—115 metres. Transmission is not permitted between 7.30 and 10.30 p.m. (18.30—21.30 G.M.T.). Through the courtesy of the Telegraph Department, the call-signs allotted will begin with the figure 7, and the letters used will, as far as possible, be the applicant's initials; this arrangement allows previously unlicensed experimenters to retain the call-signs they have hitherto used. QSL cards may be sent via Mr. James Steffensen (D7JS), Ehlersvej 8, Hellerup, Denmark.

The

By L. G.

THE pra of an a suitable is one which radio experin meter can rea work of sever in the first in The presen alters the res ment, as far cerned; a fac fully apprecia assumes an i infrequent. For really in certain typ resistance of a on different account, and article to ind the ordinary while in ne of the met ment, yet e of the circui ciably when another. Before pro posed modific instrument i reader in fo ment if a br of the sin recalled. Suppose t certain defle through it, a shunt it in s same reading flowing throu Let the resi $=r_g$, and the added $=r_s$.

Effective Resistance of Inductance Coils at Radio Frequency.—Part IV.

By S. Butterworth [R144

(*Admiralty Research Laboratory, Teddington.*)

22. Number of Turns in Coil.

A complete consideration of the factors governing design should also include the question of the relative number of turns in various shapes of coils, as it is clear that a coil of few turns is more quickly wound than one of a large number of turns. A glance at Table VIII. shows that L_0 diminishes both with increase of winding length and of winding depth.

Since $L = L_0 N^2 D$, the turns must therefore increase to hold the inductance constant as either b or t increases.

This is true if the diameter of the coil is held constant. For coils of equal surface areas the diameter diminishes as the winding length increases and this diminution must be further compensated for by added turns. These considerations give a further bias in favour of short shallow coils.

23. Determination of the Best Wire Diameter.

Whatever the basis of design we must make the wire diameter such that $R_s = MR_h$ where M = 1, 2, 3.5, or 3.67 under the various alternative conditions considered above.

If in all cases we make $R_s = R_h$, which is the correct relation for minimum resistance in a given space at high frequencies, then at low frequencies the resulting resistance is 1.06 times the least resistance possible in the available space, and 1.18 times the least resistance possible with the weight of copper employed. Thus if we use the condition of equal copper losses to determine the best diameter of the wire, we shall never be very far from the most efficient condition for the shape of coil employed. It is, however, to be understood that it may be profitable to use wire of a diameter slightly less than that estimated by the following method, particularly in the case of coils of large inductance working at long wavelengths.

Equating the values of R_s and R_h as given by equation (35) our equation to determine the wire diameter is

$$1+F = \tfrac{1}{4}(KNd/D)^2G = \tfrac{1}{4}(K^2/L_0)\,(L/D^3)d^2G$$
$$= \frac{S^2L}{D^3}d^2G = P^2d^2G \qquad (47)$$

in which we have written S^2 for the shape factor $\tfrac{1}{4}K^2/L_0$ and then put

$$P^2 = \frac{LS^2}{D^3} \qquad \ldots \qquad \ldots (48)$$

Remembering that F and G are functions of $df^{\frac{1}{2}}$, that is, of $(Pd)\,(f/P^2)^{\frac{1}{2}}$, equation (47) may be regarded as an equation connecting the two variables Pd and f/P^2, so that, if a single curve be drawn with f/P^2 as abscissæ and Pd as ordinates, this curve may be used to determine d, the only calculation necessary being that for P from equation (48).

The curve connecting Pd with f/P^2 is given in Fig. 7, together with a table of values of the shape factor S for multilayer coils. For single-layer coils the following values hold for S :—

SINGLE-LAYER SOLENOIDS.

b/D	0.125	0.250	0.375	0.500
S	1.30	0.76	0.57	0.48

SINGLE-LAYER DISC COILS.

t/D ..	0.1	0.2	0.3	0.4	0.5
S ..	1.67	1.12	1.02	1.06	1.16

In order to obtain a long range for the abscissæ the horizontal scale has been made logarithmic and the curve divided into two sections (A) and (B). The section (A) holds from $f/P^2 = 10^4$ to 10^6 and the section (B) from $f/P^2 = 10^6$ to 10^8. When $f/P^2 = 10^8$ it is seen that Pd has nearly settled down to the constant value 0.165, so that this relation may be used to find d for values of f/P^2 greater than 10^8.

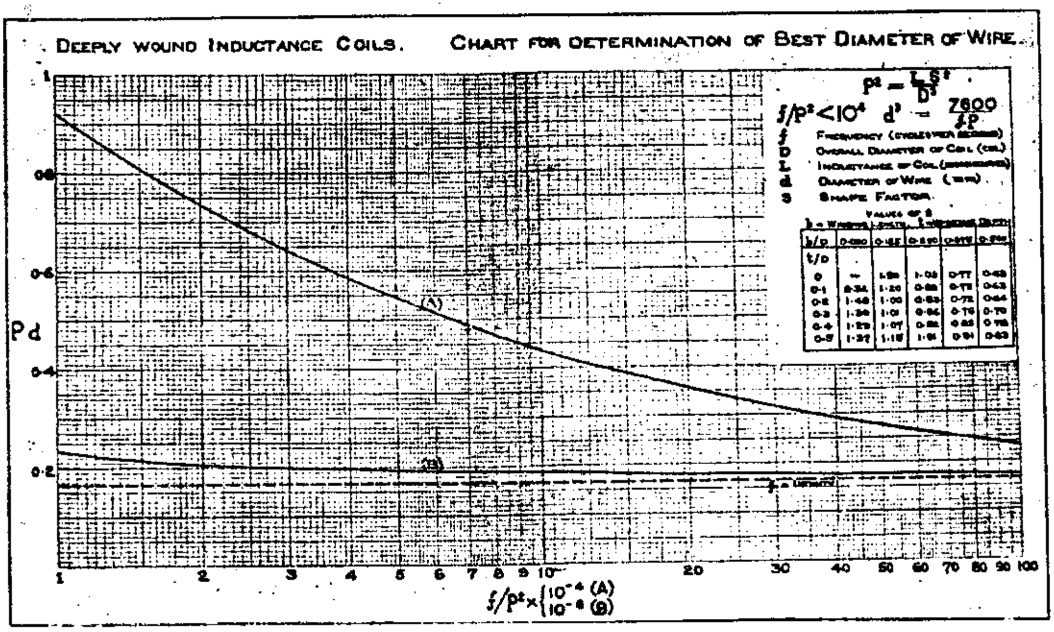

Fig. 7.

When f/P^2 is less than 10^4 the value of d is got from the formula

$$d^3 = \frac{7{,}600}{fP} \quad \ldots \quad \ldots \quad (49)$$

It will be noticed that the values of S in the Table are not equal to $K/2L_0^{\frac{1}{2}}$ but merely proportional thereto. This does not affect the accuracy of the curve provided that the constant of proportionality be included in (47) in calculating the curve. The reason for the change was to make S equal to unity for a shape which was considered to possess general efficiency, viz: $b/D = 0.125$, $t/D = 0.2$. In fact, the curve was calculated so as to give a direct relation between f and d for a coil of this shape for which $L = 1{,}000\mu H$ and $D = 10$ cms. The mode of calculating the points for the curve for this coil was to take a series of values of $df^{\frac{1}{2}}$ (thus fixing F and G) and then to find the corresponding values of d from (47). The value of f follows since $df^{\frac{1}{2}}$ is known.

24. Application to 2,000μH Coil.

In order to show the application of the theory in a practical case, we will consider the case of a coil intended to be used at a wavelength of 1,600 metres, as this is a wavelength for which most coils on the market are at present wound with the wrong diameter of wire. We will choose for the inductance of the coil 2,000μH, as at the above wavelength the resonating capacity has the convenient value of 360$\mu\mu$F. A convenient size and efficient shape (see Tables IX. and X.) is

$$D = 4 \text{ in.} = 10 \text{ cm.}, \quad b = 1.25 \text{ cm.},$$
$$t = 2.0 \text{ cm.}$$

For this shape by Table VIII. $L_0 = 0.10$, so that the number of turns required is

$$\sqrt{\frac{2{,}000 \times 1{,}000}{9.10 \times 10}} = 149$$

by equation (36). We now determine the correct gauge of wire by means of Fig. 7.

From the Table accompanying the Fig.

$S = 1$, and therefore by equation (48)

$$P^2 = \frac{2{,}000 \times 1^2}{10^3} = 2 \text{ and } P = 1.414.$$

For a wavelength of 1,600 metres the frequency is 188,000 so that $f/P^2 = 9.4 \times 10^4$. Therefore, using curve (A), $Pd = 0.445$ and this gives $d = 0.314$ mm. The nearest gauge to this is No. 30 S.W.G. Remembering that this is the gauge which will make $R_h = R_s$ and that the best condition should be such that R_h is somewhat less than R_s as the frequency cannot be regarded as "high" in association with this diameter, it will probably be profitable to use No. 32 gauge.

To verify the design and to determine the expected copper resistance the following table of values of the copper resistances of the coil, using various wire gauges, has been calculated by means of the resistance equation (35).

Wire gauge.	D.C. resistance (ohms).	A.C. resistances.		Ohms. $R_c = R_s + R_h$.
		R_s.	R_h.	
36	21.7	21.7	1.2	22.9
34	14.8	14.9	2.5	17.4
32	10.8	10.9	4.7	15.6
30	8.2	8.3	8.1	16.4
28	5.7	6.0	13.7	19.7
26	3.9	4.3	24.0	28.3
24	2.6	3.1	37.6	40.7

The Table thus verifies the design in that No. 32 wire is showing the lowest resistance. The gauges have been carried to No. 24, as many commercial coils employ gauges up to this value. When it is pointed out that the winding section can accommodate the required turns with No. 18 wire it will be seen that even with the thickest tabulated wire there is ample spacing but not enough to reduce the copper losses to their minimum value. The advocates of very thin wire are justified by the results of the Table, as No. 36 wire gives better results than the gauges usually employed.

To return to the design, all we now require is the mode of distributing the turns throughout the winding section. This distribution will hardly affect the copper losses, but will have a considerable effect upon the self-capacity and therefore on the dielectric losses (such as they are).

The principle is simple. Double spacing requires considerable care and an elaborate framework, while the gains are doubtful. With single spacing (that is spacing between the layers but none between the turns in one layer) the smallest self-capacity will be obtained if the layers are short and many. Thus with a coil of the present shape the layers must be parallel to the winding length and spaced throughout the winding depth. Allowing for insulation we should be able to obtain 30 turns in a winding length of 1.25 cm. and thus we should need five layers spaced at distances of 4 mm.

The former should have an inner diameter of about 6.4 cm., giving for the diameter of the outer layer 9.6 cm. The effective diameter is then 10 cm. as we must add the layer spacing to the outer layer diameter to obtain the effective outer diameter.

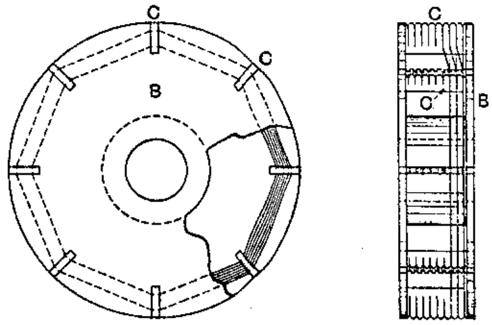

Fig. 8.

If we had chosen a shape in which the winding depth was less than the winding length, the type of winding to employ would be a "banked" winding with spacing between the banks. If suitable insulating combs are employed to support the banks a coil of this type may be wound very rapidly and thus has an advantage over a coil of the above type when very close coupling between other coils is not contemplated. The procedure of design for this type of coil is exactly as in the above case, the number of banks and of turns per bank being decided upon after the correct diameter of wire has been found.

An efficient coil of the "banked" type is shown in Fig. 8. The combs c, c . . . holding the windings are of ebonite $1\frac{1}{4}$ in. $\times \frac{1}{2}$ in. $\times \frac{1}{8}$ in. and have nine slots $\frac{3}{8}$ in. deep and $\frac{1}{8}$ in.

apart, the slot width being such as to give an easy fit to No. 30 D.S.C. wire. A V-shaped opening is filed into each slot mouth so that the wire will slip readily into its proper slot when winding. The combs are supported as shown round the circumference of a mahogany bobbin of external diameter $4\frac{3}{4}$ in., the width of the cheeks being $\frac{1}{8}$ in. and their internal separation 1 in. If the coil is intended for use at 1,600 metres the wire to use is No. 30 D.S.C. and the coil is wound bank by bank, allocating 15 turns to each bank so that the total turns in the coil is 135. Three of these banks are shown wound in the end elevation.

A coil wound by the writer to the above specification gave the following effective dimensions :—

$$D=11.0 \text{ cm.}, \quad b=2.7 \text{ cm.}, \quad t=0.65 \text{ cm.}$$

As regards D it will be noticed that the coil is octagonal, and therefore the value given for the external diameter has been taken as the mean between the diameters of the inscribed and circumscribed circles of the external octagon. The theoretical calculations assume this diameter for the equivalent circular coil.

It was found that the measured and calculated inductance were both equal to 2,420μH, the measurement being made

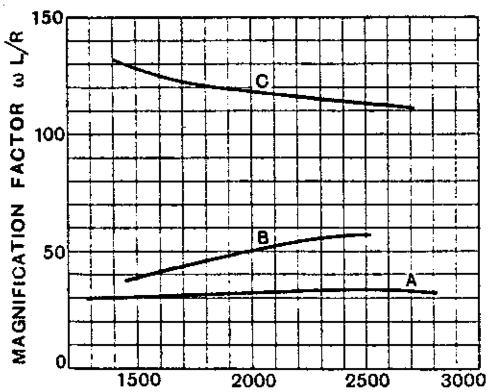

Fig. 9.

on an inductance-capacity bridge at a frequency of 1,000 cycles per second An inductance measurement made at radio frequency by the calibration department of *E.W. & W.E.* gave a value of 2,445μH and a self-capacity of 1.5$\mu\mu$F.

As regards the high frequency resistance the writer obtained the following results using the resistance variation method and a thermo galvanometer of resistance 5.1 ohms as detector. The tabulated measured values include all losses in the circuit apart from the copper losses in the leads and the D.C. losses in the detector.

The calculated resistances have been obtained from equation (35), using the measured value of the D.C. resistance, viz.: 10 ohms.

Wavelength (metres)	Obsd. A.C. Res. (ohms.)	Calcd. A.C. Res. (ohms.)
2,270	15	12
2,500	16	14
2,070	18	16
1,850	21	18
1,580	23	20
1,470	24	21

Two measurements of H.F. resistance were also made by the calibration department of *E.W. & W.E.*, in which the losses in the rest of the circuit were carefully estimated. They found at

1,605 metres A.C. resistance = 20 ohms.
2,090 ,, ,, ,, = 15 ,,

These are in practical agreement with the calculated copper resistance and thus indicate that the copper losses are the sole measurable losses in the coil.

In order to see what gains have been effected by scientific design two popular commercial coils having an approximate inductance of 2,000μH were also measured. The results are exhibited in the curves of Fig. 9. In this fig. the curves of $\omega L/R$ are plotted against wavelength for each coil. It is readily shown that if an E.M.F. e be impressed on a resonant circuit containing the coil and a condenser free from loss the P.D. developed across the condenser terminals is given by $\omega Le/R$, so that the factor $\omega L/R$ tells us what magnification of the applied E.M.F. may be obtained under ideal conditions when using the coil under test. This factor is probably thus the best factor to use for the comparison of coils of slightly different inductances.

The two commercial coils give the curves marked *A* and *B* and the above "banked winding" coil gives the curve *C*. The data for the coils *A* and *B* are as follow :—

	Coil *A*	Coil *B*
Wire gauge	24	26
External diam.	8.3	6.6 cm.
Coil length	1.5	2.6 cm.
Self-capacity	5	21μμF.
Inductance	2,300	2,180μH.

The curves show that at 1,600 metres the magnification factors for the coils *A*, *B* and *C* are 31, 42 and 125 respectively. It is true that in constructing what we consider to be the best solid wire coil for reception at this wavelength we have allowed ourselves rather more space than is usually assigned to these coils, but the difference is not so much due to this as to a correct choice of wire diameter. For coils of the size of *A* and *B* it is not unreasonable to expect a magnification factor of 100 without having recourse to stranded wire, provided that the correct diameter of wire is employed and the coils are wound in such a manner as to give the minimum self-capacity.

25. Stranded Wire Coils.

We are now to investigate what advantage may be gained by winding coils with stranded instead of solid wire. The type of stranded wire studied is that known as "litzendraht" in which each strand is separately insulated either by enamel or a single silk covering and the strands braided together in such a manner that throughout the length of the multiple conductor an individual strand occupies in turn all possible positions in the winding section. The object of this braiding is to ensure that each strand of the parallel system is subject to the same induced E.M.F. when alternating current flows along the conductor. Otherwise the outer strands would shield the inner strands and the current at high frequencies would only be carried by the outer strands. The necessary intermingling of the strands is usually brought about by constructing the cable on the "three" system, in which three strands are first twisted together to form a single cable and then three 3-stranded cables are twisted to form a 9-stranded cable and so forth so that we may use cables having 3, 9, 27, . . . 3^r strands. The wire gauges used in constructing stranded cable usually vary from No. 36 to No. 44.

It will be seen from the mode of construction that the D.C. resistance of a stranded cable having *n* strands is somewhat greater than the resistance of an equivalent length of solid conductor having the same copper section, as each strand makes a helical path and has thus a length greater than the length of the cable. This fact should be remembered when we are comparing theoretical and observed resistances.

There appears to be considerable diversity of opinion as to the relative merits of stranded and solid conductor for use in A.C. work. Some experimenters have even come to the conclusion that stranded wire coils have higher resistances than *corresponding* solid wire coils. The italics are introduced as the whole controversy on this point revolves round the fair method of comparison. The experimental comparison usually made is between coils having some simple common property in addition to equality between inductances. The favourite mode of comparison is to make the D.C. resistances equal and then to compare the A.C. resistances of the pair over a range of frequencies. When this mode of comparison is adopted it is found that the stranded wire coil has the initial advantage, the resistance of both coils increasing as the square of the frequency but the rate of increase of resistance of the solid wire coil being greater. At higher frequencies the solid wire coil falls away from the square law before the stranded wire coil, so that the two resistance curves tend to approach. At a certain frequency the curves cross and the solid wire coil then becomes better than the stranded wire coil. The experimenter then says that it is better to use solid wire above the frequency of cross over. The critical frequency he recommends depends upon the type of coil he happens to use for his experiments.

A more scientific mode of comparison was that made by Prof. Howe in a theoretical investigation on stranded conductors (*loc.cit.*[10]) Howe pointed out that for a given overall diameter of stranded cable there was an optimum number of strands of given diameter which would give a minimum A.C. resistance. He estimated the minimum resistance in a number of cases and compared it with the resistance which would be obtained with a solid conductor having the same overall diameter as the stranded wire. The cases he considered were those of straight cable and of long single layer solenoids. He described this as a "safe" mode of comparison, as we should be certain that both types of conductor would fit into the available space. It may be said that the comparisons did not augur well for stranded wire, particularly at the higher frequencies. In criticism of the method it may be pointed out that we have already shown that there is a great deal of air space still available in deeply wound inductance coils when the best diameter of solid wire is employed, so that it may (and will) turn out that the best stranded wire coil will have greater overall diameter than the best solid wire diameter and yet the wire may be accommodated. The above comparison in these cases therefore fails.

Prof. Fortescue (*loc. cit.*[11]) made a still better comparison, in that he found the best solid wire diameter and the best number of strands and compared coils under these conditions. The limitations of his theory have already been discussed and it is considered advisable to use the same method of comparison with these limitations removed.

The basis of comparison will therefore be between the best solid wire and best stranded wire coil at a particular frequency. An experimental comparison on this basis for all frequencies is obviously a task of some magnitude, as it would involve winding a great number of coils, particularly if we are ignorant as to the best diameters of wire to employ. It is better to compare the theoretical resistance formulæ with experiment in a few typical cases and, if these comparisons justify the formulæ, deduce the relative merits of the two types of coils from the theoretical formulæ.

The comparison will differ from that of Fortescue in that the best gauge for a given number of strands will be found instead of the best number of strands for a given gauge, as the latter mode of comparison often leads to a number of strands not practically available; whereas if we choose the nearest gauge to that indicated by theory for a known possible number of strands we are never very far from the minimum possible resistance.

25.1. *Resistance of Stranded Wire Coils.*

Let d be the diameter of one strand, n be the number of strands and d_o the overall diameter of the stranded wire. Then it has been shown (*loc. cit.*[12]) that the alternating current resistance of the stranded conductor when straight is given by the formula

$$R_s = R\{1 + F + k(nd/d_o)^2 G\} \quad \dots \quad (49)$$

in which R is the D.C. resistance of the stranded conductor and F and G have the same significance as before, z being calculated from the diameter of a single strand. As regards k, this depends upon n. For the usual types of stranded wire we have:—

n	=	3	9	27	large
k	=	1.55	1.84	1.92	2

For the case of a coil of stranded wire the resistance due to the remaining turns is calculated as in the case of solid wire coils. Thus, since the D.C. resistance of each strand is nR, and since there are n strands in each turn, we obtain

$$R_h = \tfrac{1}{4} R (KNnd/D)^2 G \quad \dots \quad (50)$$

Hence for the whole resistance

$$R_c = R_s + R_h = R\{1 + F + (k/d_c^2 + \tfrac{1}{4}K^2N^2/D^2)\, n^2 d^2 G\} \quad (51)$$

25.2. *Test of Formula by Comparison with Observation.*

A series of stranded wire coils, forming part of the Standard Multivibrator Wavemeter in use at the National Physical Laboratory, serve admirably to test the adequacy of the formula. The comparison will also clear up certain doubts which have been raised in regard to stranded wire.

The values of the decrements ($R/2fL$) of these coils have been given by Mr. D. W. Dye (*loc. cit.*[1]) and the coil details are to be found in the "Specification and Notes" supplied by the N.P.L. From these data the following Tables have been prepared.

The calculated A.C. resistances have been corrected for capacity in accordance with the formula already given. The agreement is very satisfactory, and shows that the losses other than copper losses are small even in these very efficient coils. The last column gives the magnification factor $\omega L/R$ and is inserted to show the increase in efficiency to be expected by resorting to stranded wire and having coils of this diameter. It is

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Frequency. Kilocycles. Per sec.
25
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40
50
60

IND
$D=$

Frequency. Kilocycles. Per sec.
50
60
80
100

interesting to notice that the first coil has not symmetrical stranding, but this does not appear to affect seriously the copper losses.

The type of insulation covering the individual strands is given and the Tables clearly show that no disadvantage is to be expected by employing enamelled instead of silk-covered strands.

COIL I.

INDUCTANCE 100,000μH, TURNS 1,024, WIRE 10/36 S.S.C.
$D=18.8$ cm., $b=5.1$ cm., $t=5.2$ cm.

Frequency. Kilocycles Per sec.	Estimated D.C. resistance. Ohms.	Measured A.C. resistance. Ohms.	Calculated A.C. resistance. Ohms.	$\omega L/R$
10	25.3	30	28	210
15		32	31	295
20		36	35	350
25		47	38	330

COIL II.

INDUCTANCE 20,000μH, TURNS 427, WIRE 27/40 S.S.C.
$D=19.5$ cm., $b=3.0$ cm., $t=5.8$ cm.

Frequency. Kilocycles. Per sec.	Estimated D.C. resistance. Ohms.	Measured A.C. resistance. Ohms.	Calculated A.C. resistance. Ohms.	$\omega L/R$
25		10.5	10.4	300
30	9.84	10.8	10.7	350
40		12.2	11.4	410
50		15.0	12.2	420
60		19.7	13.2	380

COIL III.

INDUCTANCE 5,000μH, TURNS 200, WIRE 81/40 ENAMEL.
$D=18.0$ cm., $b=1.6$ cm., $t=5.0$ cm.

Frequency. Kilocycles. Per sec.	Estimated D.C. resistance. Ohms.	Measured A.C. resistance. Ohms.	Calculated A.C. resistance. Ohms.	$\omega L/R$
50	1.47	3.1	2.8	505
60		3.8	3.3	495
80		5.4	4.8	465
100		7.0	6.7	450

COIL IV.

INDUCTANCE 995μH, TURNS 96, WIRE 81/40 ENAMEL.
$D=17.5$ cm., $b=1.5$ cm., $t=5.8$ cm.

Frequency. Kilocycles. Per sec.	Estimated D.C. resistance. Ohms.	Measured A.C. resistance. Ohms.	Calculated A.C. resistance. Ohms.	$\omega L/R$
100	0.63	1.2	1.1	520
150		1.7	1.7	550
200		2.4	2.5	520
250 ✓		3.4	3.6	460

COIL V.

INDUCTANCE 202μH, TURNS 36, WIRE 81/40 S.S.C. DOUBLE-LAYER DISC COIL.
$D=16.7$ cm., $t=3.9$ cm.

Frequency. Kilocycles. Per sec.	Estimated D.C. resistance. Ohms.	Measured A.C. resistance. Ohms.	Calculated A.C. resistance. Ohms.	$\omega L/R$
250	0.258	0.9	0.7	350
300		1.1	0.9	345
400		1.7	1.4	300
500		2.4	2.0	265
600		3.4	2.9	225

As regards Coil V, Mr. Dye stated that its decrement curve (which is distinctly worse than that for the other coils) was not understood, but the Tables show that it is behaving exactly as expected by theory. As to how it may be improved will be considered when we have dealt with the theory of the design of stranded wire coils.

25.3. *Design of Stranded Wire Coils.*

The procedure of design is exactly analogous to that for solid wire coils, using formula (51) for our resistance equation. The only difficulty is that due to the term involving d_0, the overall diameter of the stranded wire. If we regard the number of strands (n) as fixed and seek the best diameter of an individual strand, then, as d varies, d_0 will also vary, but not so rapidly as d because of the space occupied by the insulation. As a rough approximation, therefore, we take d_0 to be independent of d and to have a value which is correct for the mid gauges of wire usually used in stranded conductors. This makes $d_0{}^2 \doteq 0.07n$ when d_0 is measured in millimetres. This approximation is probably good enough for the purpose, as the term involving d_0 will usually contribute only a small amount to the total resistance. When the approximate gauge has been found using this assumption, formula (51) may then be employed, using the correct values of d_0 to calculate the resistance for a gauge or two on either side in order to discover the true best diameter.

With the assumption $d_0^2=0.07n$, the method of determining the best d is as follows:—

Calculate P from the formula

$$P^2=\sigma+n^2S^2L/D^3 \quad \ldots \quad (52)$$

in which σ is a function of n having the values

$\sigma=$ ..	0	0.9	3.3	10.4	$0.4n$
for n ..	1	3	9	27	large

The best diameter d of an individual strand is then found from Fig. 7, exactly as in the case of solid wire coils.

It must be remembered that Fig. 7 gives the diameter which will make $R_s=R_h$, and since the wire usually turns out to be thin the correct relation for minimum losses is $R_s=2R_h$. The result is that the wire gauge should be somewhat less than that estimated by the above method. If, as is often the case, we are intending the coil for a range of frequencies it is well to estimate the diameter by the above method for the *highest* frequency, as then the minimum condition will fall well inside the range of working frequency.

25.4. *Example.*

In illustration of the method of design we will take Coil V. of the wavemeter series above. An examination of the Table shows that the measured A.C. resistance is much more than double the D.C. resistance, and as the D.C. resistance is practically also the skin resistance for this gauge, we see that the general field losses are too big, which indicates thinner wire if we keep to 81 strands. This is probably impracticable on account of the difficulties of manipulation. So we will try what may be obtained with 27 strands. From the coil shape we find $S=1.34$. Inserting this and the remaining factors in (52), $P^2=67$, and as we are designing for $f=600{,}000$ $f/P^2<10^4$ and therefore the formula $d^3=7{,}600/fP$ is applicable; this gives $d=0.116$ mm. and No. 40 wire is indicated. The data are now complete for calculating the A.C. resistance by formula (51), which, with $d_o=1.4$ mm., gives $R_c=1.8$ ohms at 600,000 cycles per second. This is an appreciable gain on the value found with 81/40 wire. The resistances at the lower frequencies may readily be found as the square law of frequency holds for this case. It is found that at 250,000 cycles per sec. the A.C. resistance is 1.0 ohm, somewhat higher than that for 81/40 wire, but, taken over the whole range the coil is a better coil in regard to efficiency, and this has been obtained after sacrificing the extremely fine stranding. The example clearly shows how difficult it would be to obtain the best results from stranded wire without having recourse to theory.

25.5. *Stranded and Solid Wire Coils compared.*

The comparison between the two types of coils is most readily effected by making use of formula (52). As a first approximation we will neglect σ. This approximation is suitable particularly for coils of large inductance and for coils in which a high degree of stranding is employed. For coils otherwise equal, P is then proportional to n. Now if the frequency is *low* the best diameter of wire is given by (49), from which we see that d is proportional $1/n$. Further, when the best diameter of wire is used, the A.C. resistance is proportional to the D.C. resistance of the stranded conductor, so that both are proportional to $1/nd^2$ that is to $1/n^{\frac{1}{3}}$. For *low* frequencies, therefore, the best A.C. resistance varies inversely as the cube root of the number of strands. Next suppose the frequency so high that the square root law holds for *all* possible diameters of strand. The law for determining the best diameter is then $Pd=0.165$ and since P is proportional to n, nd is constant. But at these high frequencies the A.C. resistance is inversely proportional to nd. Thus at very high frequencies there is no gain whatever by stranding.

If, now, σ be included, the low frequency gain estimated above will be somewhat reduced and at very high frequencies stranding will actually increase the resistance. In practice, however, this case will rarely arise, and it will be found possible to design small low inductance coils for use at a wavelength of 100 metres which have lower resistance than the best corresponding solid wire coil.

The gain, however, will be small. The real objection to stranded wire coils at very high frequencies is not that the resistance cannot be reduced but that the gain is so small that it is not worth the extra expense. In order to obtain some concrete idea as to the relative merits of solid and stranded wire coils we may take the first five coils of the series of coils quoted in Section 15.

The following Table shows the theoretical magnifications for these coils when they are wound with the best solid and 9-stranded wire respectively, the wave lengths being such that each coil is in resonance with a condenser of 500$\mu\mu$F.

Magnifications of Solid and Stranded Wire Coils.

$D=8.3$ cm., $b=1.5$ cm., $t=3.0$ cm.

Coil No.	1	2	3	4	5
Inductance (μH) ..	73	183	343	765	2,170
Wavelength (metres)	362	563	786	1170	1,970
Magnification($\omega L/R$)					
Best solid wire ..	133	122	116	112	108
Best 9-stranded wire	192	198	203	208	224

It may be remarked here that the above shape of coil is not at the peak of efficiency in regard to shape. The coil of Section 24 is better in this respect and, as we have seen, gives a magnification of at least 125 at 1,600 metres. If wound with 9/38 wire (in 13 banks of 10 or 11 turns each) this magnification should be doubled.

It is not therefore unreasonable to expect a magnification of 100 for all solid wire coils and of 200 for all 9-stranded wire coils while still keeping to the usual sizes of receiving coils.

APPENDIX.

Determination of Mean Square Field over Winding Section of Inductance Coils.

(A) *Deeply Wound Coils.*

It is necessary first to find expressions for the axial and radial magnetic field components throughout the section of the coil. These components may be calculated if we know the axial and radial fields at any point on the end face of a "solid" coil, that is, a coil wound full from the centre. The writer was unable to find any published formulæ for these components, but as reliable formulæ exist for the mutual inductance between solid coils (*Phil. Mag.*, 29, p. 578, 1915) the necessary formulæ were derived from these fundamental formulæ, and tables of field values calculated. By using a system of curves drawn with the help of these Tables it is possible to determine the field at any point within the winding section of a coil which is not fully wound, as the process is merely one of addition and subtraction.

For the determination of the mean square field throughout the length of wire it is necessary to find the mean value of H^2r over the winding section, H being the field at radial distance r. Now as regards the mean value of this quantity over any cylinder co-axial with the coil, it is easily deduced by Taylor's Theorem that if b be the winding length of the coil a good approximation to the required mean value is obtained by taking the values at the three points within the winding section situated at $0.056b$, $0.250b$, and $0.444b$ respectively and finding their weighted mean, the respective weights being 5, 8 and 5. The same process may be applied in taking the mean of the means for the various elementary cylinders. The operation for determining the mean square field is thus reduced to the determination of the weighted value of H^2r at nine points in the winding section, whose positions and respective weights are as follow:—

u/b =	0.056	0.250	0.444
v/t	Weights.	Weights.	Weights.
0.112	25	40	25
0.500	40	64	40
0.888	25	40	25

Here u is the axial displacement of the point from the end plane of the coil and v the depth below its outer surface.

The values of K given in the text were found in this manner, but in order to remove any doubt in regard to the accuracy of the above approximation, the values of K were in a few cases also calculated by graphical integration using a much larger number of points. These determinations agreed with the tabulated values.

(B) *Solenoids and Disc Coils.*

From the formulæ given in the text for these cases it is easily shown that for single-layer solenoids $K^2=4uD^2/b^2$ and for single-layer disc coils $K^2=4uD^2/t^2$. As regards the m layered coils we merely replace u by $u+3.29(1-1/m^2)$.

(C) *Coils for which the Winding Section is very small compared with the Coil Diameter.*

In this case the function K becomes very large and it is preferable to write

$$H^2m = UN^2I^2/b^2$$

where b is the winding length and U is a function of t/b calculated by a method similar to that for the deeply-wound coils. When t is less than b, U has the following values:—

t/b =	0.00	0.02	0.04	0.06	0.08	0.10	0.20
U =	26.3	24.7	23.7	22.4	21.2	20.1	16.8
t/b =	0.40	0.50	0.60	0.70	0.80	0.90	1.00
U =	12.4	10.9	9.6	8.5	7.5	6.8	6.1

When t is greater than b it is only necessary to interchange b and t in the formula.

The above results are only strictly true for an infinite number of turns. The way in which the values for finite turns approach the tabular values may be illustrated by the case of a square section in which the wires are arranged in square order. For this case we have :—

Turns ..	4	9	16	25	36	64	100	Inf.
U ..	4.5	5.4	5.7	5.8	5.9	6.0	6.0	6.1

We may also use this type of coil to estimate the effect of groupings of the wires which are other than in square order. Thus we may take the case of a coil for which $t/b = 0.5$ having 54 turns. Suppose this first wound in 6 short layers each of 9 turns, the distance apart of layers being three times the distance of turns in a layer. We find $U = 10.9$ agreeing exactly with the tabular value for infinite turns. If the winding is in three long layers each containing 18 turns, the ratio of layer to turn distance being again 3 to 1, the value of U is 10.6.

These examples appear to indicate that the tabular values for infinite turns are good enough for purposes of design when the number of turns exceeds about 16 and will hold whether the wire is spaced in layers or arranged in square order. The mode of spacing affects the self-capacity rather than the copper losses.

(The author is indebted to the Admiralty for permission to publish the work relating to multilayer coils).

A Note on Transmission.

Taking Advantage of the Voltage Node. [R344.3

THE following method of feeding the high tension supply to the anode of the transmitting valve has been adopted at 5RF in order to eliminate the losses in

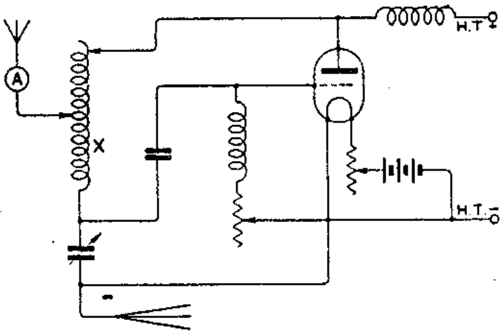

Fig. 1.

the radio-frequency choke employed in the shunt feed circuit. The removal of this choke probably has the effect also of sharpening the tuning—always a desirable feature.

In using the Colpitt's circuit, as shown in Fig. 1, it was found that a voltage node occurred at X and that the H.T. positive lead could be connected to this point on the inductance coil instead of to the anode via the radio-frequency choke.

Since the radio-frequency potential here is zero no current (except the feed current) flows along the wire W (Fig. 2) and therefore no loss of aerial current results; indeed, by this means a greater aerial current is obtained for the same power input.

The procedure for finding the nodal point is to reduce the power to one or two watts and to search along the inductance coil with the H.T. positive lead (no choke being inserted) until a point is found on either side of which the aerial current falls off. This, then, is the node where the H.T. lead is left until a change in wavelength is made. In this case, as the adjustment is critical, it will be found necessary to slide the connecting clip along the turn to which it is connected on the coil to obtain a fine adjustment. A movement of one inch makes a perceptible difference in the value of the aerial current.

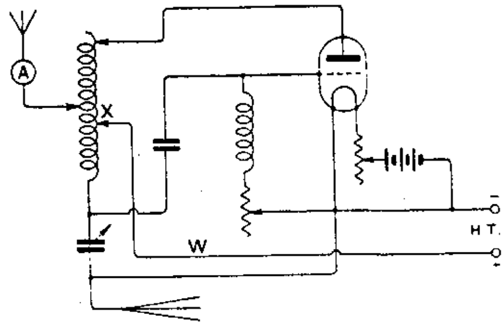

Fig. 2.

No doubt this is not a convenient method to employ where a frequent change of wavelength is necessary but it is a valuable point to consider in striving after efficiency.

L. F. HUNTER.

THE WIRELI

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