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Demagnetizing Factors of Rods

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A chart is constructed for converting the apparent magnetic permeability to the true permeability of cylinders of any given ratio of length to diameter. The curves are based on previous calculations in which account was taken of the variation of the demagnetizing factor N with permeability, but in which the permeability was assumed constant over any one cylinder. The flux distribution has been determined experimentally in several cylinders, and as the field acting on the specimen is increased from zero, the positions of the effective poles have been found to move toward the middle of the rod until (if the permeability is sufficiently high) they are located about 0.7 of the distance from the middle to the ends, and then to move toward and approach the ends as limits as the permeability declines in high fields.

WHEN using the ballistic galvanometer for making magnetic measurements on cylinders or rods it is common practice to determine directly the "apparent" permeability μ', equal to the quotient of the flux density B and the applied field strength H'. The B is determined at the middle of the rod and H' is calculated simply from the current flowing through the magnetizing solenoid. Then μ' is converted to the (true) permeability μ by calculation using the appropriate demagnetizing factor N, thus taking account of the field strength produced at the middle of the rod by the rod itself. One purpose of this note is to provide a simple graphical means of converting true to apparent permeability or *vice versa* when the specimens are cylindrical rods of any ratio of length to diameter. The chart shown has already found considerable use in practical problems dealing with such rods.

Other purposes of this report are to record experiments on the distribution of magnetization in several rods, to show how the positions of the poles change with permeability and to compare these findings with theory.

GRAPHICAL CONVERSION OF μ TO μ'

The demagnetizing factor N is given by

$$H=H'-NI$$

or the equivalent

$$H=H'-(N/4\pi)(B-H),$$

in which H is the (true) field strength, H' the applied field strength equal to that which would exist if the specimen were removed, I the intensity of magnetization and B the flux density measured at the middle of the rod. The N considered here may be termed the ballistic demagnetizing factor to distinguish it from the magnetometric demagnetizing factor which is not multiplied by the I at the middle but by some kind of an average of the intensity of magnetization over the whole bar depending on the position of the magnetometer needle.

The equation given above may be rewritten:

$$\frac{1}{\mu-1}=\frac{1}{\mu'-\mu'/\mu}-\frac{N}{4\pi}$$

or

$$\frac{1}{\mu}\left(1-\frac{N}{4\pi}\right)=\frac{1}{\mu'}-\frac{N}{4\pi},$$

the apparent permeability being defined as $\mu'=B/H'$. In practice sufficient accuracy is always obtained by using the simpler equation:

$$1/\mu=(1/\mu')-(N/4\pi).$$

It is thus possible to plot μ against μ' for a cylindrical rod of given dimensional ratio provided N is known, even if N varies with permeability. Such a plot is shown in Fig. 1. The values of N have been taken from the compilations of Stäblein, Schlechtweg, Neumann, and Warmuth[1] and are the results of calculations

[1] H. Neumann, Wiss. Veröff. Siemens-Konz. **10**, 55–71, No. 2 (1931); H. Neumann and K. Warmuth, Wiss. Veröff. Siemens-Konz. **11**, 25–35, No. 2 (1930); F. Stäblein and H. Schlechtweg, Zeits. f. Physik **95**, 630–46 (1935); K. Warmuth, Arch. f. Elektrotech. **30**, 761–79 (1936); **31**, 124–30 (1937) and **33**, 747–63 (1939).

that agree within experimental error with the experimental determinations. The curves of Fig. 1 are corrected for variation of N with μ, the permeability being that corresponding to the B existing at the middle of the rod. The original calculations of N were based on the assumption that μ is constant over the rod and this may in certain circumstances cause a definite error in the calculated μ *vs.* μ' relation.

The values of $N/4\pi$ appropriate for $\mu=\infty$ are plotted, in the upper left-hand portion of Fig. 1, against the dimensional ratio, m = length/diameter, of the rod. Since $\mu'=4\pi/N$ when $\mu=\infty$, the scale-reading at the right (for μ') is the reciprocal of the scale-reading at the left (for $N/4\pi$) for any given horizontal line (ordinate). Also, the asymptote for any given μ *vs.* μ' curve (m = constant) lies on the same horizontal line as the value of $N/4\pi$ that corresponds to this value of m.

The method of using the chart is obvious. Interpolation for values of m not given may be

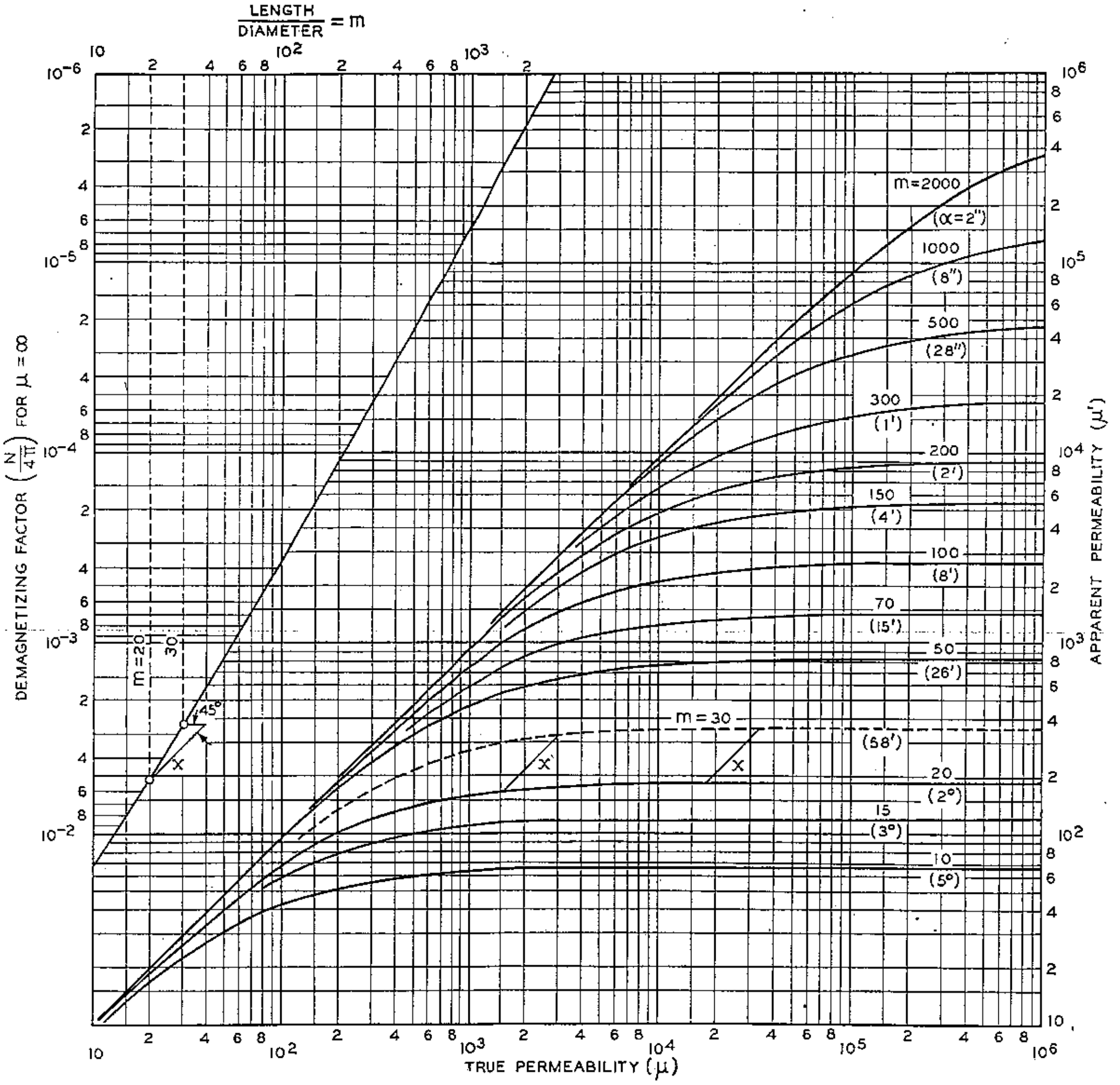


FIG. 1. Relation between apparent permeability μ' and true permeability μ of cylinders having any given ratio m of length to diameter. Demagnetizing factors N as defined by $H=H'-NI$, are shown as a function of m for rods having very high permeabilities. The curves may also be used for converting μ' to μ for ring specimens in which there is an air gap subtending an angle α at the center of the ring.

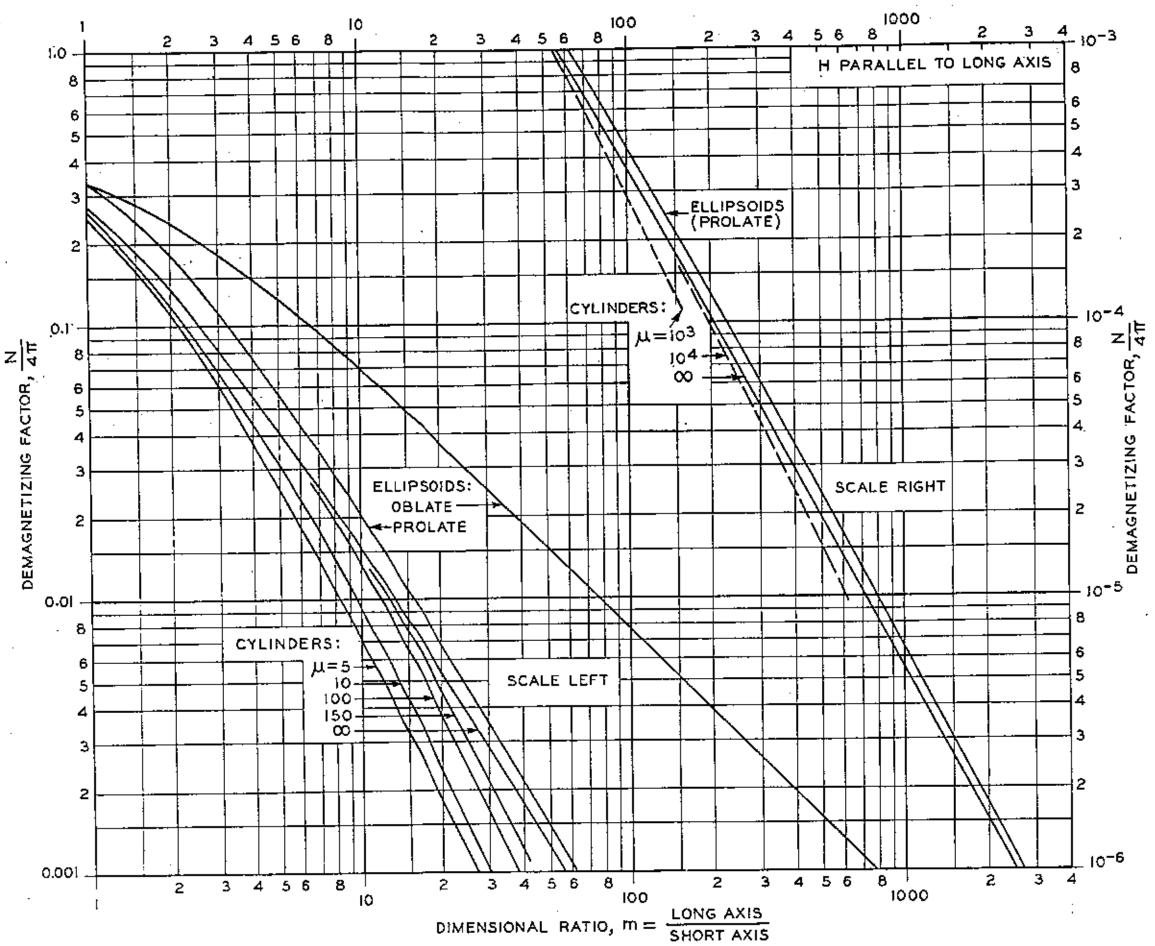


FIG. 2. Demagnetizing factors of ellipsoids and cylinders (see the equations and reference 1).

performed as follows: The appropriate value of m (say, 30) is located on the scale at the top and the corresponding value of $N/4\pi$ (0.0027) found on the curve of $N/4\pi$ *vs.* m. The value of $N/4\pi$ is noted also for a nearby value of m for which a μ *vs.* μ' curve is drawn (e.g., $m=20$). Horizontal lines are drawn through the points corresponding to these values of m and $N/4\pi$, and the 45° distance x between these lines is determined. The line x of the same length and 45° inclination, may then be drawn from any point on the curve for $m=20$ and will give a point on the curve for $m=30$, as shown by the dotted construction lines on the chart. This method of interpolation is based on the fact that all of the μ *vs.* μ' curves are similar, except as influenced by the variation of N with μ', and differ only by a translation along a line inclined 45° to the axes. The amount of such a displacement at $\mu=\infty$ is obviously the same, on a logarithmic scale, as the displacement made when a point on the $N/4\pi$ *vs.* m curve is moved from one value of m to another; therefore, this is the displacement for all parts of the μ *vs.* μ' curve. The slight correction in the interpolation for variation of N with μ is at all times negligible.

For convenience the demagnetizing factors for oblate and prolate ellipsoids of revolution, and for rods having finite permeabilities, are plotted in Fig. 2. Convenient forms of the equations for accurate calculation of $N/4\pi$ for the ellipsoids are as follows:

(1) For oblate ellipsoid with axes $a=c/m$, $b=c$,

(a) Magnetized parallel to a long axis, b or c,

$$\frac{N}{4\pi}=\frac{1}{2}\left[\frac{m^2}{(m^2-1)^{\frac{3}{2}}}\arcsin\frac{(m^2-1)^{\frac{1}{2}}}{m}-\frac{1}{m^2-1}\right],$$

$$\frac{N}{4\pi}=\frac{\pi m-2}{4m^2}\quad\text{if}\quad m\gg 1.$$

(b) Magnetized parallel to the short axis, a,

$$\frac{N}{4\pi}=\frac{m^2}{m^2-1}-\frac{m^2}{(m^2-1)^{\frac{3}{2}}}\arcsin\frac{(m^2-1)^{\frac{1}{2}}}{m},$$

$$\frac{N}{4\pi}=1-\frac{\pi}{2m}\quad\text{if}\quad m\gg 1.$$

(2) For prolate ellipsoid with axes $a=mc$, $b=c$,

(a) Magnetized parallel to the long axis, a,

$$\frac{N}{4\pi}=\frac{1}{m^2-1}\left\{\frac{m}{2(m^2-1)^{\frac{1}{2}}}\ln\frac{m+(m^2-1)^{\frac{1}{2}}}{m-(m^2-1)^{\frac{1}{2}}}-1\right\},$$

$$\frac{N}{4\pi}=\frac{1}{m^2}(\ln 2m-1)\quad\text{if}\quad m\gg 1.$$

(b) Magnetized parallel to a short axis, b or c,

$$\frac{N}{4\pi}=\frac{1}{2}\left\{\frac{m^2}{m^2-1}-\frac{m}{(m^2-1)^{\frac{3}{2}}}\ln\frac{m+(m^2-1)^{\frac{1}{2}}}{m-(m^2-1)^{\frac{1}{2}}}\right\},$$

$$\frac{N}{4\pi}=\frac{1}{2}\left(1-\frac{\ln 2m}{m^2}\right)\quad\text{if}\quad m\gg 1.$$

Neumann and Warmuth[1] give the following formula for calculating $N/4\pi$ for long cylindrical rods having infinite permeability:

$$(N/4\pi)_{\mu=\infty}=(4.02\log_{10}m-0.92)/(2m^2)\qquad m\gtrsim 10.$$

It should be remembered that the demagnitizing factors, N_1, N_2, and N_3, in three mutually perpendicular directions in a specimen of any shape add up to 4π:

$$N_1+N_2+N_3=4\pi.$$

From this it follows that the curves of Fig. 2 approach $N/4\pi=1$ as an asymptote as m approaches zero. Also it is convenient to remember that for a thin ring with an air gap that subtends the angle α at the center of the ring, the demagnetizing factor is given simply by

$$N/4\pi=\alpha/360$$

if α is in degrees. Thus for each rod of dimensional ratio m, there is a value of N that applies also to a ring having an air gap of angular width α. In Fig. 1 these values of α are recorded in parentheses just below the corresponding values of m, and the figure may thus be used for ring specimens as well as rods.

FLUX DISTRIBUTION

In addition to a knowledge of the true permeability of a rod, as determined from H' and from B as measured in a short search coil placed at the middle of a bar, it is often desired to determine this μ when the search coil covers a considerable portion of the rod or even is longer than the rod. This can be done if the distribution of flux in the rod is known. Flux distribution was determined in two rods having widely different permeabilities but the same diameter (0.97 cm), length (50.6 cm), and dimensional ratio (52.4). Both rods were cut from the same stock of molybdenum permalloy containing 79 percent nickel, 4 percent molybdenum, and the rest iron; one was heat-treated for high permeability while the other was unannealed, as swaged.

Measurements were made with a fluxmeter using a small search coil 2 mm long placed at various positions along the rod. The applied field of given strength H' was reversed in direction to produce the required change in flux. The results are shown in Figs. 3 and 4 in which the flux density B_l at any point along the rod is referred to the density B at the middle. Actually flux-densities are corrected by subtracting H' from each of them; they are in reality B_l-H' and $B-H'$. The abscissa l/l_0 is the position of the coil on the rod expressed as the fraction of the distance from the middle to the end. Each curve is marked with the corresponding value of H' and corresponding values of μ' and B are given in the insert.

The distribution of flux in the annealed rod when the permeability is highest ($H'=5.0$, $\mu'=871$) is nearly parabolic. For comparison, the dotted line of Fig. 3 is drawn as a parabola cutting the experimental curve at $B_l/B=0.5$. The nearby dashed line is the distribution accurately calculated by Würschmidt[2] under the assumption that $\mu=\infty$; it is nearly parabolic.

The position of the effective pole, given by

$$l_p/l_0=\int_0^\infty \frac{B_l-H'}{B-H'}d\frac{l}{l_0},$$

may be determined* for each of these theoretical curves and for the experimental curves; for the parabolic distribution it lies at $l_p/l_0=0.73$, Würschmidt's distribution gives 0.73, and for these experimental curves l_p/l_0 lies between 0.72 and 0.96.

The demagnetizing factor of a rod depends not only on the dimensional ratio and on the permeability, if that is uniform throughout the bar, but also on the variation of permeability if, as is generally the case, it is different in one portion of the rod than in another. It has been reported[3] that the positions of the effective poles move toward the middle of the rod and then outward again as the magnetization is increased from zero, and it has been pointed out[4] that they are nearest the middle when the magnetization is approximately that corresponding to maximum permeability. The more recent calcu-

[2] J. Würschmidt, *Theorie des Entmagnetizierung Factors und der Scherung von Magnetizierungskurven* (Vieweg, Braunschweig, 1925).

* This integral is equal to

$$\int_0^1 \frac{B_l-H}{B-H'}d\frac{l}{l_0} \quad \text{since} \quad \int_0^\infty (H_l-H')d(l/l_0)=0$$

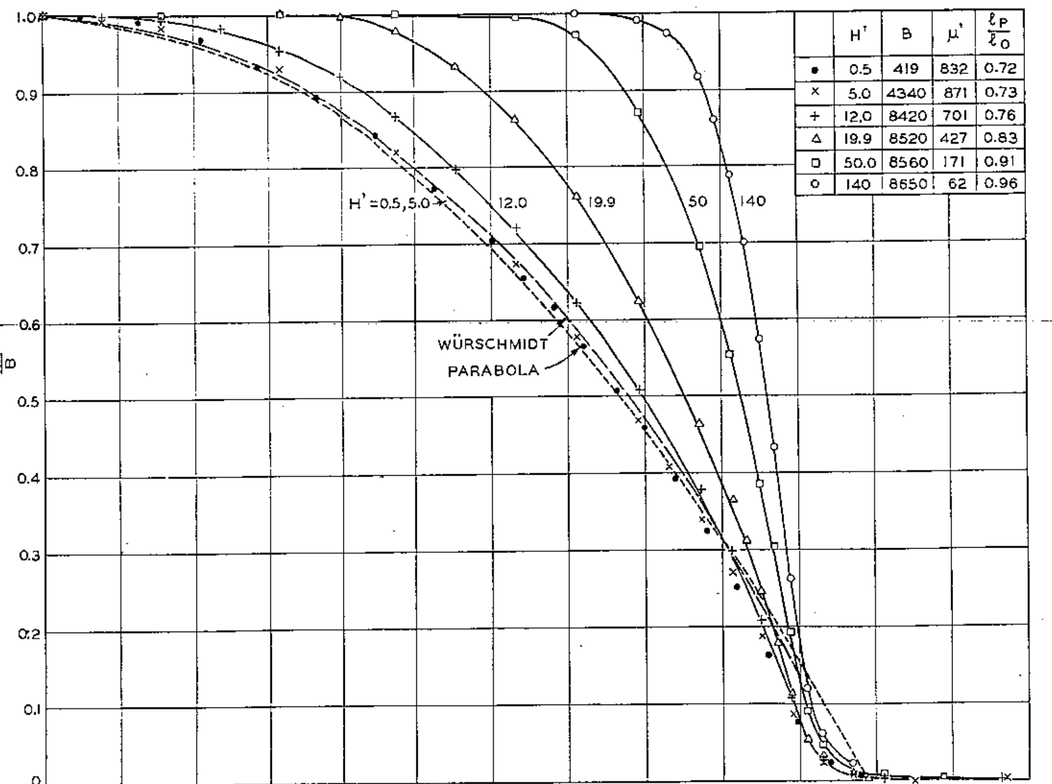
and

$$\int_1^\infty (B_l-H)d(l/l_0)=0,$$

as pointed out to us by Mr. H. J. Williams. In all of our experiments $B-H$ and $B-H'$ (both at the middle of the rod) are indistinguishable.

[3] L. Holborn, Sitz, Preuss. Akad. Wiss. 159–68 (1898); C. G. Lamb, Phil. Mag. **48**, 262–71 (1899).

[4] G. F. C. Searle and T. G. Bedford, Phil. Trans. Roy. Soc. **A198**, 33–104 (1902). For further discussion of change of N with B, see E. Dussler, Ann. d. Physik **86**, 66–94 (1928) and D. Foster, Phil. Mag. **8**, 304–13 (1929).



	H'	B	μ'	ℓP/ℓ0
•	0.5	419	832	0.72
×	5.0	4340	871	0.73
+	12.0	8420	701	0.76
△	19.9	8520	427	0.83
□	50.0	8560	171	0.91
○	140	8650	62	0.96

FIG. 3. Distribution of flux in rod of annealed permalloy having dimensional ratio $m=52.4$. Values of B, H', μ' and l_p/l_0 (pole position) are indicated in the table. Theoretical distribution (broken line) for $\mu=\infty$ is as calculated by Würschmidt.

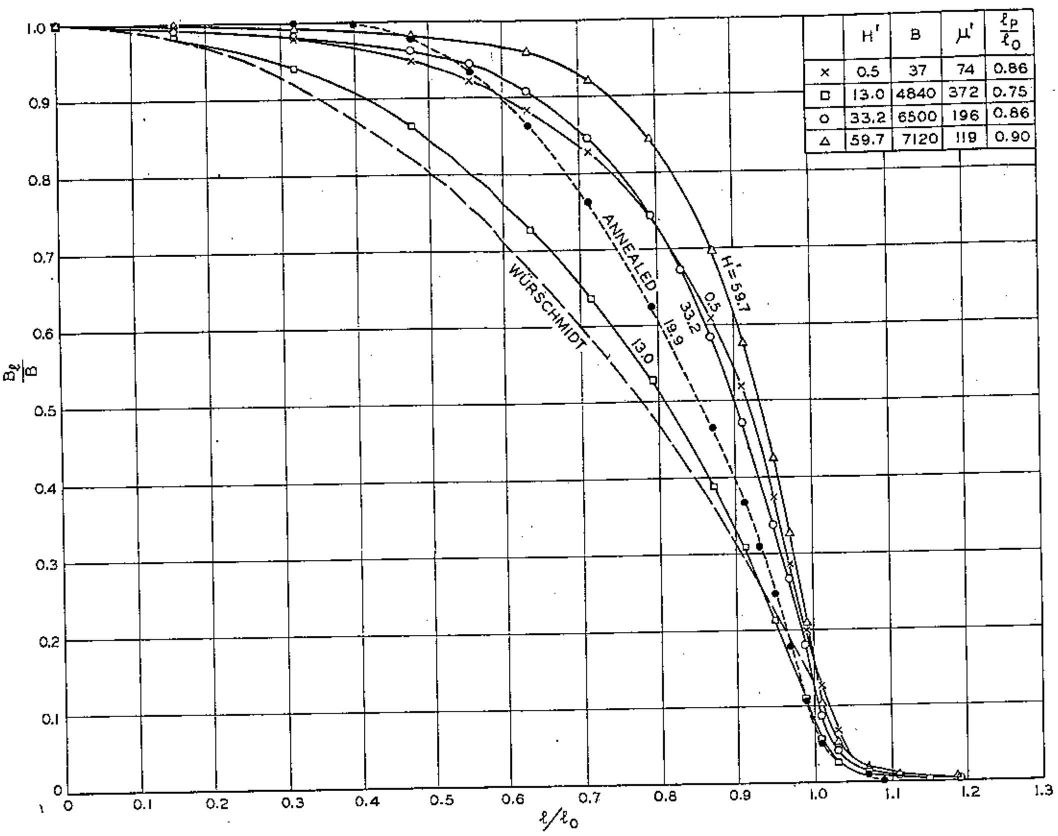


FIG. 4. Distribution of flux in rod of unannealed permalloy ($m=52.4$). B, H', μ' and l_p/l_0 are as indicated. Würschmidt's curve (broken line) and an experimental curve for the annealed rod ($H=20$) are shown for comparison.

lations of Würschmidt establish the fact that a high demagnetizing factor and a small interpolar distance are associated with high permeability. Thus, theory and experiment justify the following description of the movements of the effective poles as the applied field is increased from zero. The poles appear first at some position, between $l_p/l_0=0.7$ and $l_p/l_0=1$, determined by the true initial permeability μ_0 of the rod. As H' increases μ also increases in all ferromagnetic materials and the poles, therefore, move toward the middle of the bar, approaching the limiting position $l/l_0 \doteq 0.7$ the more nearly the higher the maximum permeability, μ_m. As H' increases further and μ declines from μ_m to 1, the poles move toward the ends of the rod and approach these ends as limits. The positions of the poles are given in the table inserted in Figs. 3 and 4.

When the permeability is well below the highest value the distribution is far from parabolic, as is evident from the curves. In the lowest fields the permeability of the unannealed rod is low and uniform, and it is possible to calculate the flux distribution and compare with experiment. Such calculations could be made as described by Stäblein and Schlechtweg[1] but they are laborious and have not been carried out for this report. When the applied field-strength is increased the permeability at the middle is greater than that at the ends; the poles are then nearer the middle than they would be if the permeability of the whole bar were that of the middle, for the low permeability at the ends effectively shortens the rod. That this is so is indicated by the data for the annealed rod for $H'=0.5$; in this case the poles were already near their closest-in positions because of the high permeability, and the increase in μ at the middle causes the poles to move in still farther to $l_p/l_0=0.72$, a minimum value for this rod and a value less than that corresponding to Würschmidt's calculated distribution for $\mu=\infty$.

When either rod is subjected to an intense field, the distribution near the middle is naturally quite uniform because the field over a considerable portion of the rod is sufficient to cause saturation. The distribution curve for $H'=20$ for the annealed rod may be contrasted with that for $H'=0.5$ for the unannealed rod. The former curve shows only a very slight dropping of the flux in going from the center to $l/l_0=0.4$, and this is due to the fact that this rod is practically saturated while the unannealed rod has an almost uniform permeability over its entire length.

Attention may be called to the very slight differences in distribution in the unannealed rod when $H'=0.50$ and 33.2. This may be attributed to the relatively slight change in permeability in different parts of the rod, as compared to the large range in permeabilities in the annealed rod.

It seemed desirable to establish for rods of all dimensional ratios the apparent fact that the effective pole has a minimum interpolar distance given by $l_p/l_0 \doteq 0.7$. To this end a wire of 70 permalloy was heat treated in a magnetic field[5] to develop high permeability and its effective pole positions were measured. The wire was 30.4 cm long, 1.01 mm in diameter ($m=300$), its effective permeability $\mu'=18{,}250$,* and the pole positions were found to be $l_p/l_0=0.70$. Thus, the minimum pole separation corresponding to $l_p/l_0 \doteq 0.7$ may be assumed to apply to all cylinders with very high permeability. A rod may be considered to have "very high permeability" for this purpose, if its μ is so high that increasing it does not increase the μ' of the rod by more than 10 or 20 percent as indicated by the curves of Fig. 1.

[5] J. F. Dillinger and R. M. Bozorth, Physics 6, 279–84 (1935).

* Using Neumann and Warmuth's Eq. (12) of reference 1, $N/4\pi=0.00005021$ for $\mu=\infty$ and $m=300$. Combining this with our $\mu'=18{,}250$ gives $\mu=218{,}000$, a value consistent with previous results for this material, reference 5.

LENGTH OF SEARCH COIL

The distribution curves can now be used to calculate the way in which the flux threading the search coil varies with the coil's length. In practical measurement this amounts to being able to determine B at the middle of the bar from the galvanometer deflection made with a search coil of any length. The calculation is simply the averaging of B_l over the length of the search coil, and results are shown in Fig. 5

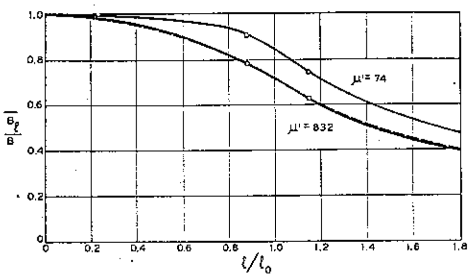


FIG. 5. Effect of length of search coil on measurement of B. Search coil (of length $2l$) and rod (of length $2l_0$) are concentric. B is flux density at middle, $\bar{B}_l$ is the average flux density in that part of the rod that is "covered" by the search coil. The dashed line is calculated for the parabolic distribution of Fig. 3.

for the two molybdenum permalloy rods in fields of $H'=0.5$. Search coils of two lengths, one 88 percent of the length of the rod and one considerably longer than the rod, were constructed and points on the curves were checked as indicated in the figure. The dotted line is calculated for the parabolic distribution curve shown in Fig. 3. It is evident that for a rod of dimensional ratio near $m=50$, an effective permeability of $\mu'=700$ ($\mu \doteq 3000$) is sufficiently low to cause an appreciable departure from the curve for the nearly parabolic distribution associated with infinite permeability. For the upper curve of Fig. 5, $\mu'=74$ and $l_p/l_0=0.86$.