CONDENSED VERSION OF BUTTERWORTH'S PAPER

The Effective Resistance of Inductance Coils at Radio Frequency*

An Abstract by B. B. Austin, B.Eng., A.M.I.E.E., of a Paper by S. Butterworth

1.0.—Introduction

IN 1926 *The Wireless Engineer*, then known as *Experimental Wireless* published a most complete treatise by S. Butterworth on the "Effective Resistance of Inductance Coils at Radio Frequency." This treatise was published in four separate issues and occupied some thirty-two pages. The subject was dealt with very completely and the sources of the various losses carefully analysed.

The writer recently wished to make a considerable number of H.F. coil calculations and therefore referred to this treatise.

It soon became apparent however that it would not be possible to pick out the necessary formulae immediately and make the desired calculation. In fact the whole treatise had to be carefully read through and notes made of the parts relevant to the work in hand; this required quite a number of hours work. The paper was thus condensed into a few pages.

As it is some time since the original paper appeared, it is thought that the republication of it in this abstract form will be of interest to readers. *No original work has been done*, but in one or two of the tables extensions have been made to cover coil shapes not given and which the writer required.

2.0.—General

The circular Coil being the most general shape in use, consideration is limited to this type of Coil. A multi-sided Coil (*i.e.*, one with not less than six sides) may be taken as equal to a circular Coil the diameter of which is the mean between the inscribed and circumscribed circles of the polygon.

For convenient reference a list of symbols used has been included at the end of the article.

Design Procedure

3.1.—Influence of Coil Shape

The shape of the Coil has considerable influence on the H.F. resistance. It can be shown that the best shape of all is a single layer solenoid in which the winding length is equal to one-third of its diameter. The best single layer disc Coil should have a

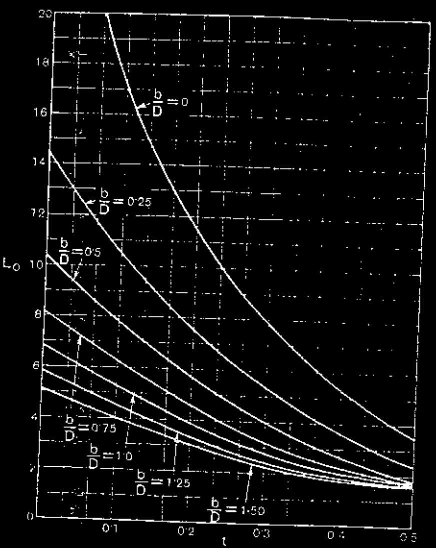

Fig. 1.

winding depth equal to one quarter the external diameter. For a multi-layer Coil there is a wide range of choice all of which would be equally efficient. If $5t + 3b = D$ the condition for maximum efficiency is never very greatly departed from.

D = External Diam. t = winding depth.
b = winding length.

3.2.—Influence of Wire Diameter

The losses in a Coil at high frequencies may be divided into three parts:—

(*A*) The copper loss which would occur if the wire were straight: this loss decreases with increase of the diameter of the wire. It is denoted in the following by R_s.

(*B*) The increase of copper loss due to the wire being coiled. This loss increases with increase of wire diameter. It is denoted in the following by R_b.

(*C*) Dielectric losses. These are not likely to be serious for wavelengths above 300 metres if ordinary precautions are taken. Experiments show them to be 30–40 % of the total loss at 300 metres with coils wound on solid wood formers, while with carefully chosen dielectric the loss is from 10–20 % of the whole.

From (*C*) above it will be seen that at 300 metres the calculated value of H.F. resistance may be taken as 80 % of the total.

From (*A*) and (*B*) it will be seen that for a given shape and inductance of Coil, as the diameter of wire is increased one source of copper loss will increase and the other decrease. There is therefore a certain

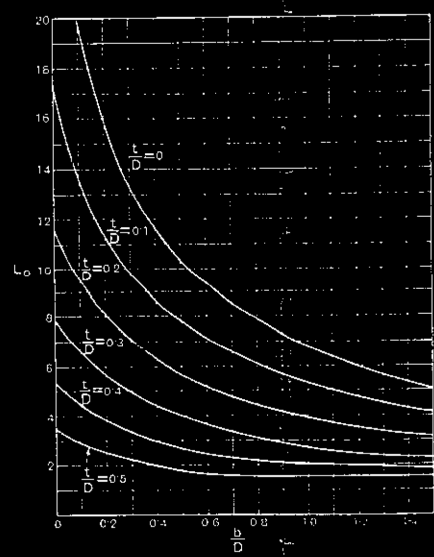

Fig. 2.

diameter of wire at which the rate of increase of one loss will equal the rate of decrease of the other, and at this diameter the total loss will be a minimum.

3.3.—Calculation of Inductance

As Coils have always to meet an inductance requirement it is necessary to be able to determine inductance for any Coil shape.

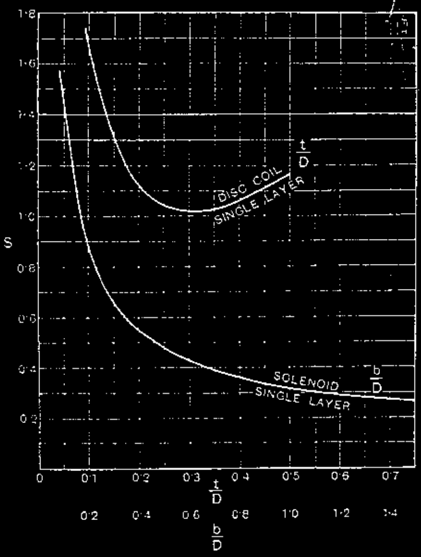

Fig. 3.

Usually this means calculating the number of turns for a pre-determined shape.

The inductance of a coil may be written as:

$$L = L_0 N^2 D/1000 \quad \quad (1)$$

where N = number of turns, D = external diam. (cm.), L = inductance in $\mu H.$ and L_0 is the shape factor given by Figs. 1 and 2. Fig. 1 has $\frac{b}{D}$ as parameter and Fig. 2 has $\frac{t}{D}$ as parameter.

3.4.—Calculation of Best Wire Diameter. (Solid Wire.)

Knowing the Coil shape and number of turns the best wire diameter must now be found.

The sources of loss were indicated in paragraph 3.2. It can further be shown that if the wire diameter is chosen so that $R_s = R_b$ for any type of Coil, the least loss will never be greatly departed from. It may sometimes be an advantage to use a

B

* MS. accepted by the Editor, February, [illegible]

wire of slightly less diameter than that given by the following method. It is however an easy matter to calculate the H.F. resistance for a gauge or two on either side of that cal-

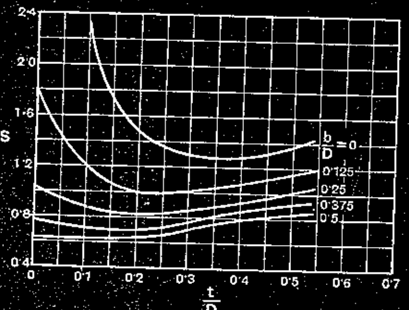

Fig. 4.

culated, and take the lowest. The value of P can be found from the expression:

$$P^2 = \frac{LS^2}{D^3} \quad \cdots \quad \cdots \quad \cdots \quad (2)$$

where S is found from Figs. 3 and 4. The value of $\frac{f}{P^2}$ can now be determined and the value of Pd read off from the curve of Fig. 5. The value of "d" is now known. In the above if $\frac{f}{P^2} < 10^4$ "d" is obtained from the expression $d^3 = \frac{7{,}600}{fP}$ and when $\frac{f}{P^2} > 10^8$ "d" settles down to the constant value of 0.165.

3.5.—Calculation of Best Wire Diameter. (Stranded Wires.)

The usual gauge of the separately insulated strands is between 36 and 44 S.W.G. The procedure is to calculate the best

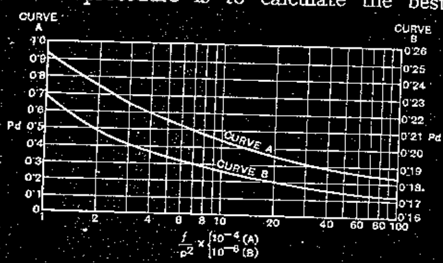

Fig. 5.

diameter of a single strand. The value of P is slightly modified and is found from the expression

$$P^2 = \sigma + \frac{n^2 S^2 L}{D^3} \quad \cdots \quad \cdots \quad (3)$$

in which σ is a function of "n" and has the following values:

$n = 1 \quad 3 \quad 9 \quad 27 \quad$ large (No. of strands)
$\sigma = 0 \quad 0.9 \quad 3.3 \quad 10.4 \quad 0.4n$

TABLE.

VALUES OF THE FUNCTIONS F AND G.

d = diameter of wire (cm.); ρ = resistivity (cgs units f = frequency cps; $Z = \pi d \sqrt{\frac{2f}{\rho}}$. For copper of resistivity 1,700 cgs units $d\sqrt{f} = 9.28$ or $Z = .1078 d\sqrt{f}$.

Z.	1 + F.	G.	Z.	1 + F.	G.
0.0	1.000	—	5.2	2.114	0.790
—	—	—	5.4	2.184	0.826
0.1	1.000	—	5.6	2.254	0.861
0.2	1.000	—	5.8	2.324	0.898
0.3	1.000	$Z^4/64$	6.0	2.394	0.932
0.4	1.000	—	—	—	—
0.5	1.000	0.00097	—	—	—
0.6	1.001	0.00202	6.2	2.463	0.967
0.7	1.001	0.00373	6.4	2.533	1.003
0.8	1.002	0.00632	6.6	2.603	1.038
0.9	1.003	0.01006	6.8	2.673	1.073
1.0	1.005	0.01519	7.0	2.743	1.109
1.1	1.008	0.02196	7.2	2.813	1.144
1.2	1.011	0.03059	7.4	2.884	1.180
1.3	1.015	0.04127	7.6	2.954	1.216
1.4	1.020	0.0541	7.8	3.024	1.251
1.5	1.026	0.0691	8.0	3.094	1.287
1.6	1.033	0.0863	8.2	3.165	1.322
1.7	1.042	0.1055	8.4	3.235	1.357
1.8	1.052	0.1265	8.6	3.306	1.393
1.9	1.064	0.1489	8.8	3.376	1.428
2.0	1.078	0.1724	9.0	3.446	1.464
2.1	1.004	0.1967	9.2	3.517	1.499
2.2	1.111	0.2214	9.4	3.587	1.534
2.3	1.131	0.2462	9.6	3.658	1.570
2.4	1.152	0.2708	9.8	3.728	1.605
2.5	1.175	0.2949	10.0	3.799	1.641
2.6	1.201	0.3184	11.0	4.151	1.818
2.7	1.228	0.3412	12.0	4.504	1.995
2.8	1.256	0.3632	13.0	4.856	2.171
2.9	1.286	0.3844	14.0	5.209	2.348
3.0	1.318	0.4049	15.0	5.562	2.525
3.1	1.351	0.4247	16.0	5.915	2.702
3.2	1.385	0.4439	17.0	6.268	2.879
3.3	1.420	0.4626	18.0	6.621	3.056
3.4	1.456	0.4807	19.0	6.974	3.233
3.5	1.492	0.4987	20.0	7.328	3.409
3.6	1.529	0.5160	21.0	7.681	3.586
3.7	1.566	0.5333	22.0	8.034	3.763
3.8	1.603	0.5503	23.0	8.388	3.940
3.9	1.640	0.5673	24.0	8.741	4.117
4.0	1.678	0.5842	25.0	9.094	4.294
4.1	1.715	0.601	30.0	10.86	5.177
4.2	1.752	0.618	40.0	14.40	6.946
4.3	1.789	0.635	50.0	17.93	8.713
4.4	1.826	0.652	60.0	21.46	10.48
4.5	1.863	0.669	70.0	25.00	12.25
4.6	1.899	0.686	80.0	28.54	14.02
4.7	1.935	0.703	90.0	32.07	15.78
4.8	1.971	0.720	100.0	35.61	17.55
4.9	2.007	0.738	—	—	—
5.0	2.043	0.755	Large	$(Z\sqrt{2}+1)/4$	$(Z\sqrt{2}-1)/8$

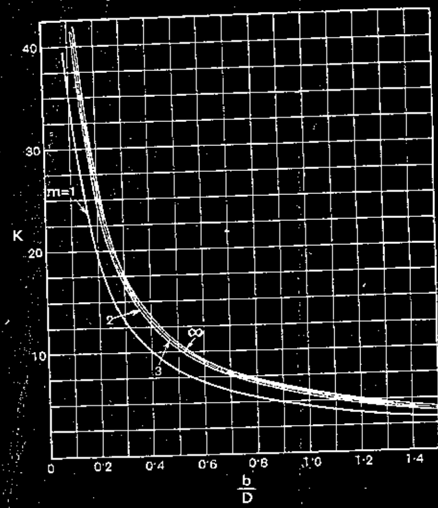

Fig. 6.

The best diameter "d" of an individual strand is then found from formula 2 and Figs. 3 and 4 exactly as for a solid wire.

3.6.—Calculation of H.F. Resistance. (Solid Wire.)

The formula for the combined copper losses may be written:

$$R_c = R_s + R_h = R\left\{1 + F + \tfrac{1}{4}\left(\frac{KNd}{D}\right)^2 G\right\} \quad (4)$$

Where

R = D.C. resistance of coil.
$1 + F$ & G. are found from the Table.
D = External diam. of coil cms.
d = Diameter of wire cms. (Not including insulation.)
N = Number of turns.
K = Is found from Figs. 6, 7 and 8.

For Coil shapes other than single layer, disc or multi-layer, see S. Butterworth's paper, appendix.

3.7.—Calculation of H.F. Resistance. (Stranded Wire.)

As the best type of stranded wire contains 3, 9, 27 . . 3^r strands, the number of strands which are to be used is first settled. A practically available number of strands is thus always chosen. Formula 4 is now modified and becomes

$$R_c = R_s + R_h = R\left\{1 + F + \left(\frac{k}{d_0^2} + \frac{K^2N^2}{4D^2}\right)n^2 d^2 G\right\} \quad \cdots \quad (5)$$

where

n = number of strands.
d = diam. of one strand cms.
R = D.C. resistance of stranded conductor.
d_0 = overall diam. of conductor in mm.
$= \sqrt{.07n}$ (see following paragraph.)

k is a function of n as follows:—

$n = \quad 3 \quad 9 \quad 27 \quad$ large
$k = \quad 1.55 \quad 1.84 \quad 1.92 \quad 2$

A slight difficulty occurs in calculating the value of d_0 the overall diameter of the stranded conductors. As a rough approximation therefore d_0 is taken as independent of d and is given by

$$d_0^2 = 0.07n \quad \cdots \quad \cdots \quad \cdots \quad (6)$$

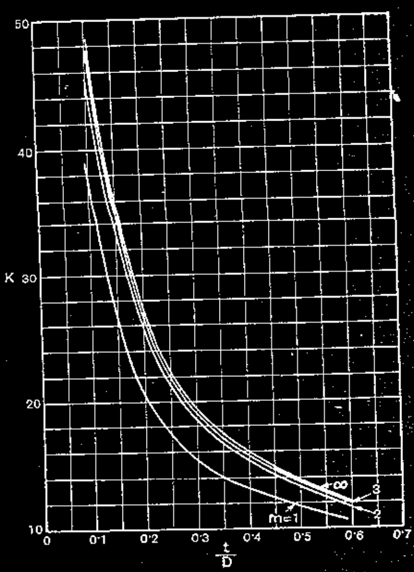

Fig. 7.

This is nearly correct for gauges between 36 and 44 S.W.G.

In general the best wire diameter will be slightly less than calculated by paragraph 3.5. Therefore, where a Coil is required to cover a range of frequencies, the best wire diameter should be found for the highest frequency, then the condition of minimum resistance will fall inside the range of the working frequency. Alternatively the H.F.

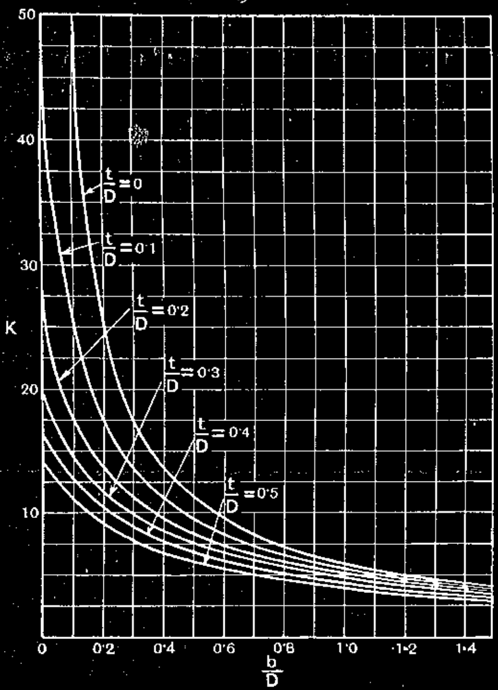

Fig. 8.

resistance may be found for a gauge or two on either side of that calculated and that gauge chosen which gives the least resistance over the working range.

3.8.—H.F. Resistance of a Straight Wire

For a straight wire the formula becomes

$$R_c = R_s = R(1 + F) \quad \cdots \quad (7)$$

where $1 + F$ is found from the Table.

3.9.—List of Symbols

D = Diameter of Coil in cms.
b = Winding length of coil in cms.
t = Winding depth of coil in cms.
R_s = Copper loss in a straight wire.
R_h = Increase in Copper loss due to coiling the wire.
R_c = Total Copper loss in ohms.
L = Inductance in microhenries.
L_0 = An inductance factor found from Figs. 1 or 2.
N = Number of turns in coil.
P = A factor found from equation 2.
f = Frequency in cycles per second.
d = Diameter of solid wire or a single strand in cms. except in Fig. 5 where it is in mm.
n = Number of wires in a stranded conductor.
S = Shape factor found from Figs. 3 or 4.
σ = Factor depending on number of strands.
R = D.C. resistance of conductor in ohms.
K = Factor given in Figs. 6, 7 or 8.
k = Factor in Formula 5 depending on number of strands.
d_0 = Overall diameter in mm. of stranded conductor (see Formula 6).

The Physical Society's Exhibition

THE Twenty-fourth Annual Exhibition of Scientific Instruments and Apparatus, arranged by the Physical Society, will be held on January 9th, 10th and 11th, 1934, at the Imperial College of Science and Technology, Imperial Institute Road, South Kensington, S.W.7. The Sessions will be as follows:—

Tuesday, January 9th, 1934 (admission by ticket only), 3 p.m. to 6 p.m. and 7 p.m. to 10 p.m.

Wednesday, January 10th, 1934 (admission by ticket only), 4 p.m. to 6 p.m. and 7 p.m. to 10 p.m.

Thursday, January 11th, 1934 (admission without ticket), 3 p.m. to 6 p.m. and 7 p.m. to 10 p.m.

The leading manufacturers of scientific instruments will be exhibiting their latest products in the Trade Section. The Research and Experimental Section will contain contributions from most of the important research laboratories in Great Britain, and there will be a special sub-section devoted to experiments of educational interest. In addition, the work submitted for the Craftsmanship Competition by apprentices and learners will be on view.

Discourses will be delivered each day at 8 p.m as follows:—

January 9th—R. S. Whipple, M.I.E.E., F.Inst.P., "The Evolution of the Galvanometer."

January 10th—J. Guild, A.R.C.S., D.I.C., F.Inst.P., "The Instrumental Side of Colorimetry."

January 11th—Sir Ambrose Fleming, D.Sc., F.Inst.P., F.R.S., "The History and Development of the Thermionic Valve."

Members of Institutions and Scientific Societies may obtain tickets from their Secretaries; tickets may also be obtained direct from the Exhibition Secretary, 1, Lowther Gardens, Exhibition Road, S.W.7.

7/10

Dear Dave,

I found this very old photostat of what is apparently a classic article by Austin which makes sense of Butterworth's famous paper. I also enclosed the article from the Radiotron Designers handbook (one of the all time classics) since it refers to the same papers. I find the inductance equation from the Bureau of Standards paper to be very accurate even with rough data on "L" and "C". They also have a R_{AC}/R_{DC} equation, which predates Butterworth. I am now getting your newsletter.

Regarding the Ground Conductivity letter by Ian Drummond (June 95) I am just measuring bulk conductivity. I have used spacings which are close enough so that dielectric constant is not a problem. Since I am really just measuring null depth for a given spacing (It gets shallower as conductivity rises) I tend to overestimate conductivity unless I am very careful. The loop is electrostatically shielded, but not balanced, which seems to be OK as I get symmetrical null depths (both sides of the loop) of > 70 dB. Frequency is 3496 Hz and bandwidth is 1 Hz. All I am looking for is ballpark numbers so that I can predict the performance of 2-way voice radios. The reality is, of course, is that the S/N of the received signal underground is enhanced by the rock (compared to free space) although the signal levels are reduced. The only practical limitation is the thermal noise of the loop. Voice downlinks with CW uplinks (using the locating beacon) give comparable ranges. The problem as always is the voice uplink which is atmospheric or EMI noise limited. I have worked out ranges and differences between up and downlink performance for different depths and conductivities, incl. the effect of atmospheric noise and on that basis am going to use about 40 KHz which is clear for several KHz either side in the U.S. for my first attempt at a long range radio.

Brian Pease